Phase Noise
In
Oscillators

Kevin Aylward B.Sc.
www.kevinaylward.co.uk

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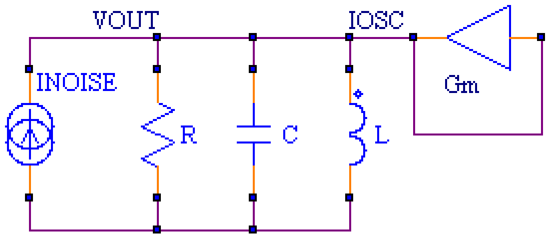
Overview

Many descriptions of phase noise in oscillators lack clear and explicit explanations as to the exact cause of phase noise. Specifically, there is much confusion as to how low frequency amplitude noise becomes up-converted to phase noise around the oscillator frequency. Furthermore, although the general application of one well known analysis, the Hajimiri-Lee approach, has been rigorously proven to be flawed, it is still sometimes promoted as a useful point of view.

Introduction

The usual starting point of an analysis of phase noise in oscillators is with Leeson's model. This model is empirical, and simply constructed as an attempt to justify already known results. It has no calculating ability whatsoever, other than modelling the general bode plot shape of the noise curve.

The key part of the model assumes all oscillators have a dominate characteristic described by an RLC tuned tank.



For frequencies close to the oscillator frequency, the impedance of the LCR tank is given by:

$$Z(\Delta\omega) = \frac{R}{1 + j2Q_l \dfrac{\Delta\omega}{\omega_0}}$$

For oscillation the loop gain must equal unity at resonance, hence:

$V_0 Gm = \dfrac{V_0}{R}$, hence

$$GmR = 1$$

To determine the effect of injected noise, the closed loop gain with respect to the noise signal is required.

V_0 due to I noise is therefore:

$$V_{0n} = (V_{0n}Gm + I_n)Z$$

$$V_{0n} = \frac{I_n Z}{GmZ - 1}$$

$$V_{0n} = \frac{I_n \dfrac{R}{1 + j2Q_l \dfrac{\Delta\omega}{\omega_0}}}{\dfrac{GmR}{1 + j2Q_l \dfrac{\Delta\omega}{\omega_0}} - 1}$$

$$V_{0n} = \frac{I_n R}{1 - (1 + j2Q_l \dfrac{\Delta\omega}{\omega_0})}$$

$$V_{0n} = -j \frac{R}{2Q_l} \frac{\omega_0}{\Delta\omega} I_n$$

At zero carrier offset, the model predicts infinite output voltage. This is essentially due to the divide by zero of the transfer function of the feedback loop. Resolving this issue requires that the limiting nature of real oscillators to be accounted for.

At high frequencies, real oscillators have additional wideband noise due to various factors. Additionally, the model simply assumes what the noise current is. Hence, these effects can be included in the above model by simply forming a vector sum with this noise, with a suitable empirical scale factor:

$$L(\Delta\omega) = F(1 + (\frac{1}{2Q_l} \frac{\omega_0}{\Delta\omega})^2)$$

And to further account for the empirical fact that low frequency noise gets up-converted, an additional factor is also included:

$$L(\Delta\omega) = F(1 + (\frac{1}{2Q_l} \frac{\omega_0}{\Delta\omega})^2 (1 + \frac{\omega_f}{(\Delta\omega)}))$$

It should be noted here that simple additive noise generates phase noise of ½ the value of the additive noise.

A more mathematically rigorous approach shows that the main noise response is Lorentzian. That is, of the form:

$$L(\Delta\omega) = \frac{K}{K + \Delta\omega^2}$$

A similar result is achieved by feeding a noise current into a parallel RLC tank circuit.

Hajimiri-Lee Model – Up Conversion

The Hajimiri-Lee Model is a model that, attempts to explain phase noise, and *very specifically*, up conversion of low frequency noise, by the proposal that it there is an alleged time variant nature, to an otherwise, essentially, linear system. This model is referred to as a LTV system rather than that of a linear model regarded as a LTI. Hajimiri-Lee (HL) make several rather major claims of the model’s correctness and explanatory power, despite these claims proving to being incorrect. HL also make claims that alternative concepts produce incorrect results, when some some of these actually do give the correct results.

The Hajimiri-Lee model is flawed for the following reasons:

1 It fails to correctly account for any frequency/phase modulation up-conversion generated by non-linear capacitances and non-linear time constants. As all real oscillators exhibit such effects, it is also a fatal flaw of the model.

2 It predicts infinite noise at the oscillator frequency.

3 Various researchers have shown that the model is invalid for flicker (up-converted) noise and non-stationary noise, despite the HL paper claiming otherwise.

4 The model can only be applied to specific oscillator types, despite claiming to be a general theory. For example, its model collapses for quadrature oscillators.

5 A. Demi[1], with reference to the HL model, mathematically proves and states:

5.1 Is the orthogonal decomposition valid in general?

5.2 Even if it is not strictly valid, can it provide approximately correct results and intuition for practical oscillator designs?

5.3 We show that the answer to both questions is negative.

5.4 ...it can predict results off by as much as 50 dBc/Hz.

5.5 ...the argument [HL] in their “proof” is flawed, and the result they “proved” is invalid.

5.6 … The right-hand-side (RHS) of the differential equation (13) for the phase error is non-linear. Thus, one can not use superposition to calculate the phase error due to several perturbations, i.e., one can not calculate the phase errors due to two perturbations separately and then sum them up to obtain the phase error due to the two perturbations applied at the same

time.

6 The root cause of up-conversion noise in real oscillators, as will be shown, is non-linearity. The Hajimiri-Lee model specifically denies this, which makes it therefore, essentially, useless in producing an optimum design that minimises up-conversion of 1/f noise.

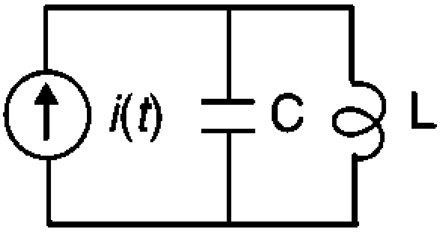
With the wealth of available information as to the non-validity of the HL model, it is still somewhat interesting that so many references still refer to the method in some positive manner.

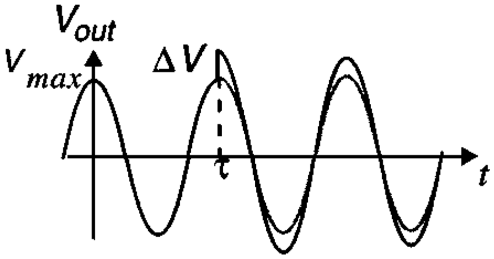
The Hajimiri-Lee Model

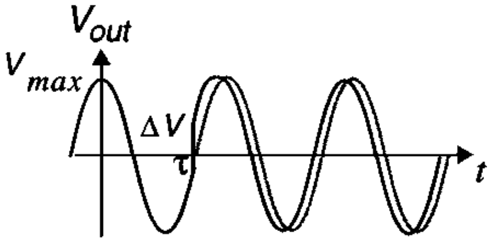
The original paper should be referred to where necessary[2].

The summary of the Hajimiri-Lee model as follows:

An LC oscillator is excited by a noise current pulse.







An argument is then presented that the phase of the oscillator system is, typically maximally changed near the zero crossings of the oscillator, and never changed at the oscillator peaks. It is also argued that effect of the current pulse will persist indefinitely.

The essentials are that the Hajimiri-Lee model re-classify a system from an assumed linear, time invariant system with one noise input signal and one oscillator input signal, with a system block consisting of a linear a lossless LC network, to that of an equivalent, alleged, time variant system, with one noise input signal driving a block containing a “hidden” oscillator signal with the linear LC circuit. A mathematical argument is then used that produces the following result for the phase of the oscillator output signal:

$$\phi(t) = \frac{I_0 c_0 \sin(\Delta\omega t)}{2 q_{\max} \Delta\omega}$$

calculated from what HL define as an Impulse Sensitivity Function (ISF):

$$\Gamma(\omega_0 \tau)$$

and this result would then directly imply that low frequency noise signals would be up-converted.

On a technical point, it should be noted that if instead of a cos() noise input, a sin() noise input is assumed, The result for phase noise becomes:

$$\phi(t) = \frac{I_0 c_0 \cos(\Delta\omega t)}{2 q_{\max} \Delta\omega}$$

This shows infinite noise at the oscillator frequency.

<u>Overview of the HL approach</u>

Consider a linear system in the Laplace transform domain.

$$\bar{v}_0(s) = \bar{v}_i(s)h(s)$$

Transforming to the time domain is achieved by

$$V_0(t) = \text{inverse laplace}[v_i(s)h(s)]$$

However, it can be shown, that if the Laplace transforms of the individual product terms are known, then the output can be directly computed from the convolution integral:

$$V_0(t) = \int_0^t V_i(\tau)H_v(t-\tau)d\tau$$

dimensionally,

$H \Rightarrow \frac{V_{out}}{V_{in}}$ so that the integral calculates

$$V_0 \Rightarrow \frac{V_{out}}{V_{in}}V_i$$

Although, the above completely specifies an output as a function of time, irrespective of any "phase" parameter, the HL model makes an assumption of an additional independent variable, the output "phase" of $V_0(t)$. The obvious extension to the above convolution integral for this assumption would then be:

$$\phi_0(t) = \int_0^t \phi_i(\tau)H_\phi(t-\tau)d\tau$$

where $\phi_i(t)$ is an input phase variable and

where $H_\phi(t)$ a transfer function relating output phase to an input phase variable:

$$H_\phi(t) \Rightarrow \frac{\phi_{out}}{\phi_{in}}$$

The equations HL write are

$$\phi_0(t) = \int_{-\infty}^t I_i(\tau)H_\phi(t,\tau)d\tau$$

$$H_\phi(t,\tau) = \frac{\Gamma(\omega_0\tau)}{q_{\max}}u(t-\tau)$$

where dimensionally,

$$H_\phi \Rightarrow \frac{\phi_{out}}{I_{in}}$$

so that

$$\phi_0 \Rightarrow \frac{\phi_{out}}{Iin}I_i$$

and that therefore H_ϕ is formulated as an output phase resulting from an input current.

So, if $\Gamma(\omega_0\tau)$ is non-zero, the output voltage automatically has an inherent inbuilt phase variation dependant on another input signal.

For technical reference, it should also be noted that there appears to be an unstated argument that, as $\Gamma(\omega_0\tau)$ is supposed periodic, the integration is independent of convolution folding.

There are some mathematical inconstancies in the HL papers, in particular Reference [2], p.183, equation 15 integrating cos() from negative infinity results in a meaningless result, despite the result actually stated.

A question immediately arises from this result of up-conversion, in that that nothing has physically changed in the system from its equivalent linear, time invariant system.

Mathematically reformulating a system cannot change its basic physics. A LTI system, remodelled as a LTV system cannot result in the existence of a something that can be proven to not exist in the original formulation. Two signals driving a linear, time invariant network cannot produce frequencies that are not present in the driving signals, no matter how the system is reformulated mathematically.

Therefore why does the HL model, seem to imply phase modulation for such a LTI system?

Cadence Simulations

A Cadence steady state simulation (PSS) for a linear RLC circuit was performed to illustrate this point.

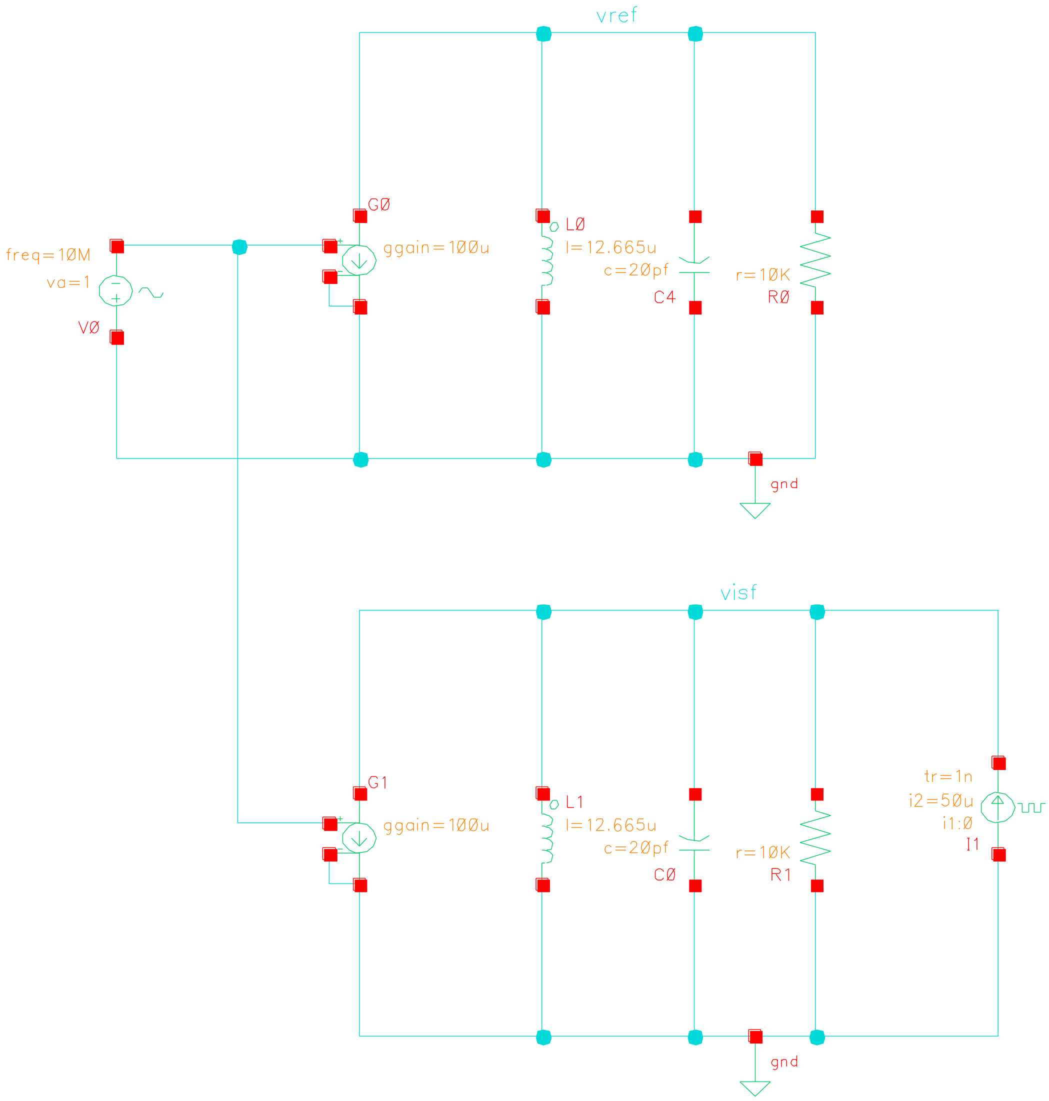
Linear System ISF

A main sine wave was generated with added square impulses at t=0 and t=π/2.

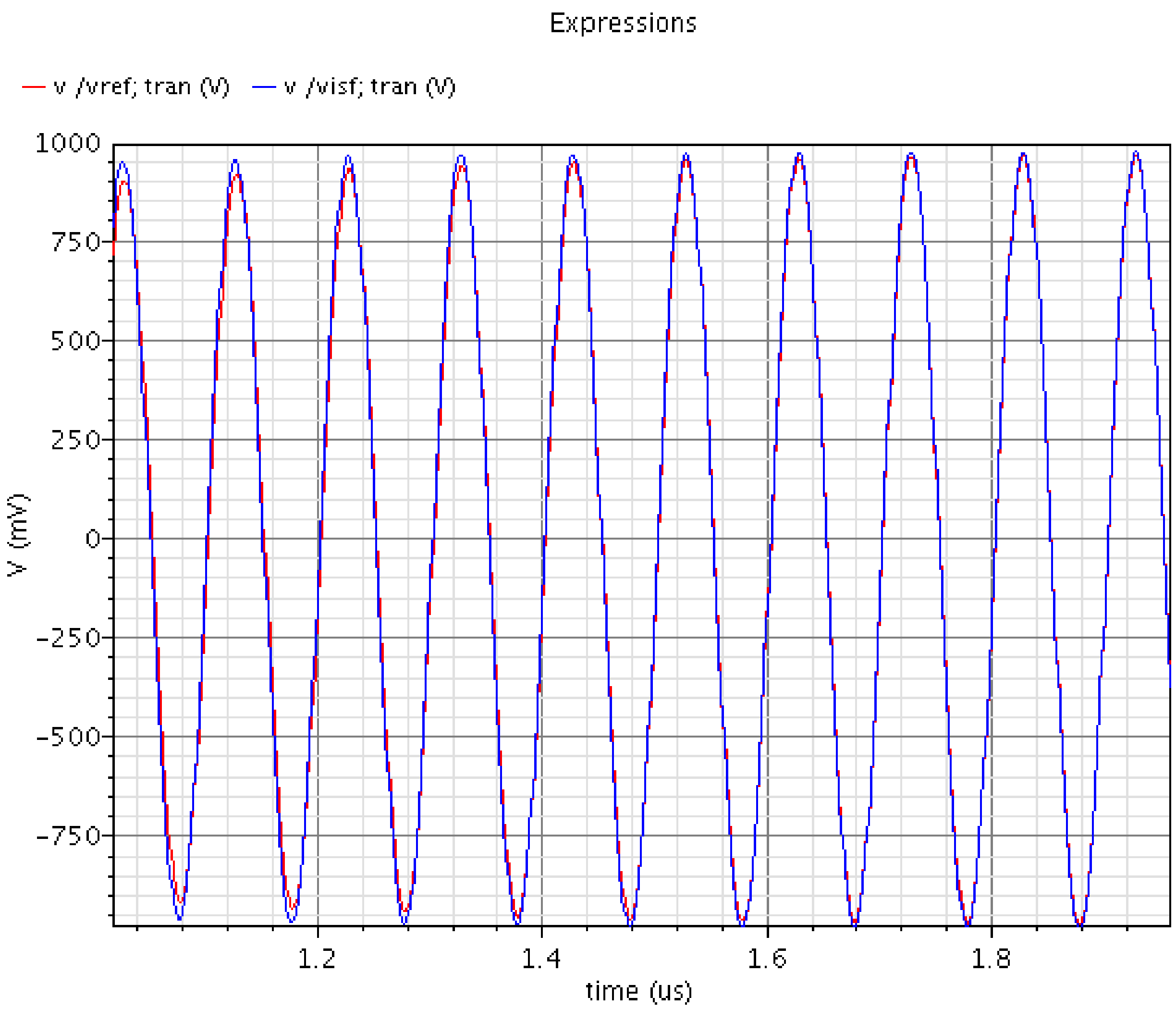
F main=10Mhz, i=100ua

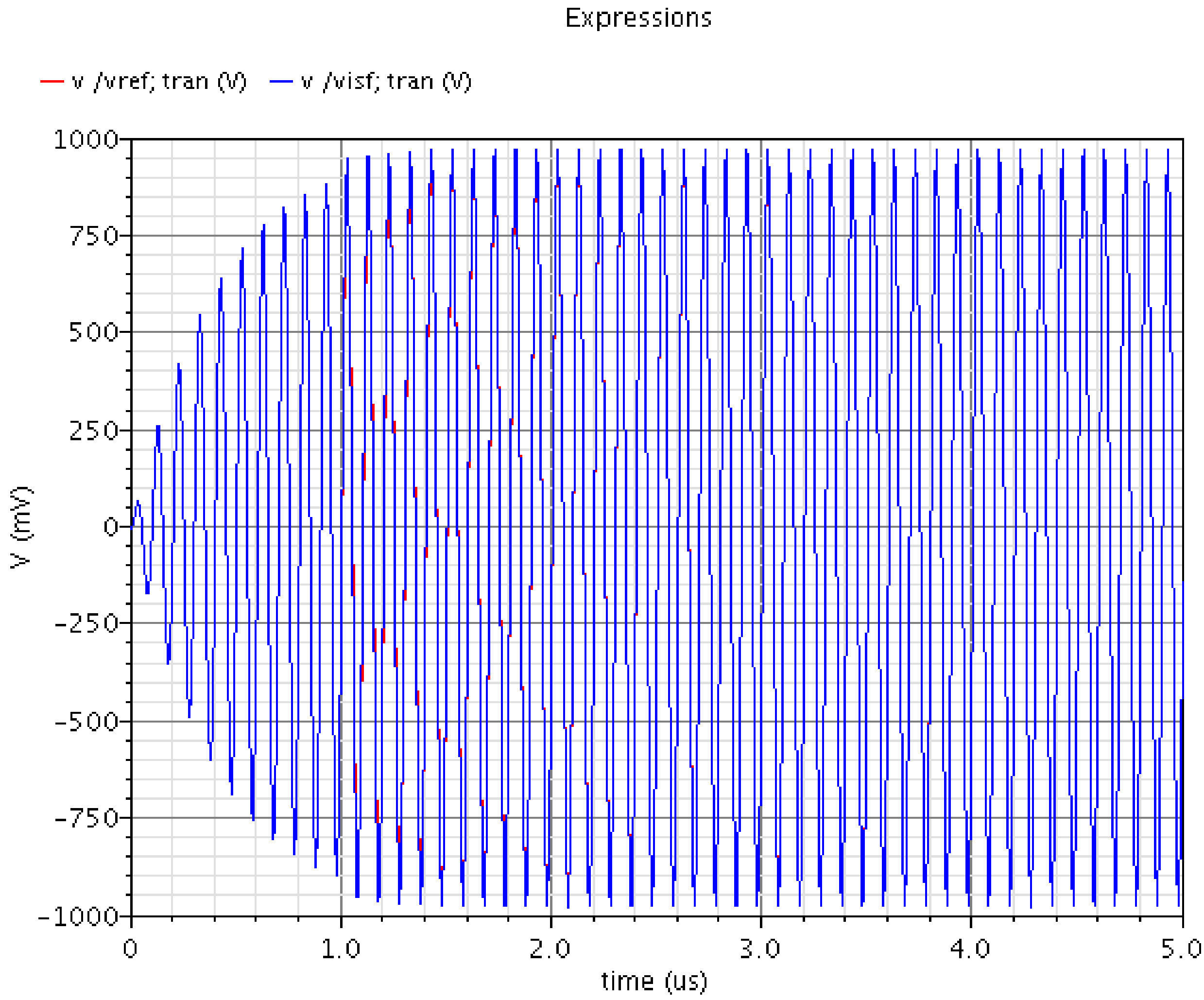
F impulse=10Mhz, PW=20ns, TR=TF=1ns, i=100ua

Linear ISF Schematics



Linear ISF Waveforms


Expressions
v /vref; tran (V)
v /visf; tran (V)
1000
750
500
250
0
-250
-500
-750
V (mV)
1.2
1.4
1.6
1.8
time (us)

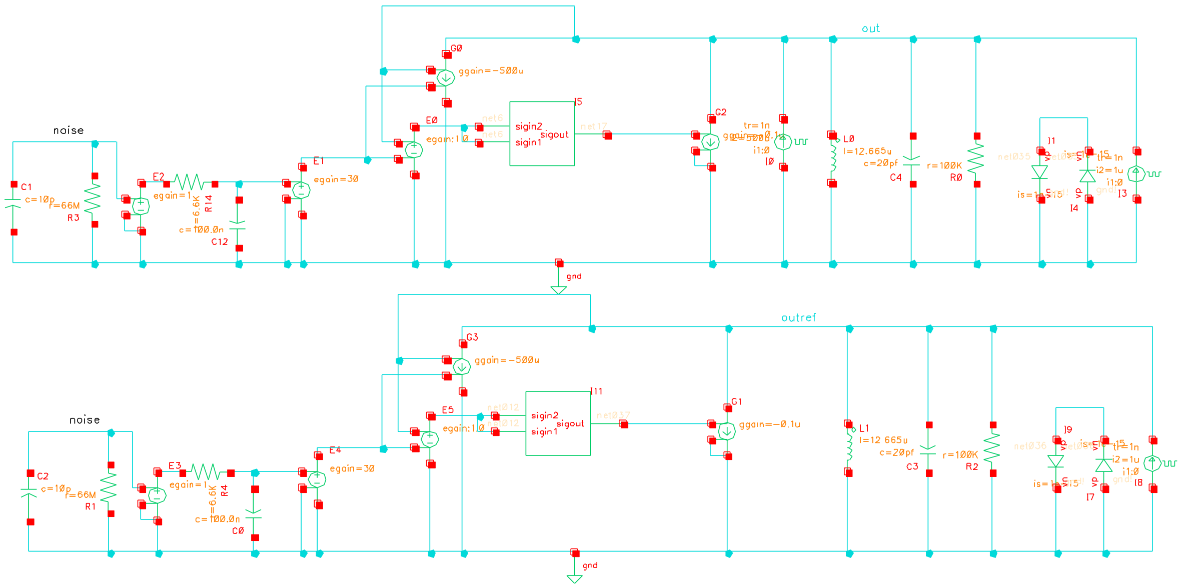


where it can be seen (red) that the phase change is only of a transient nature.

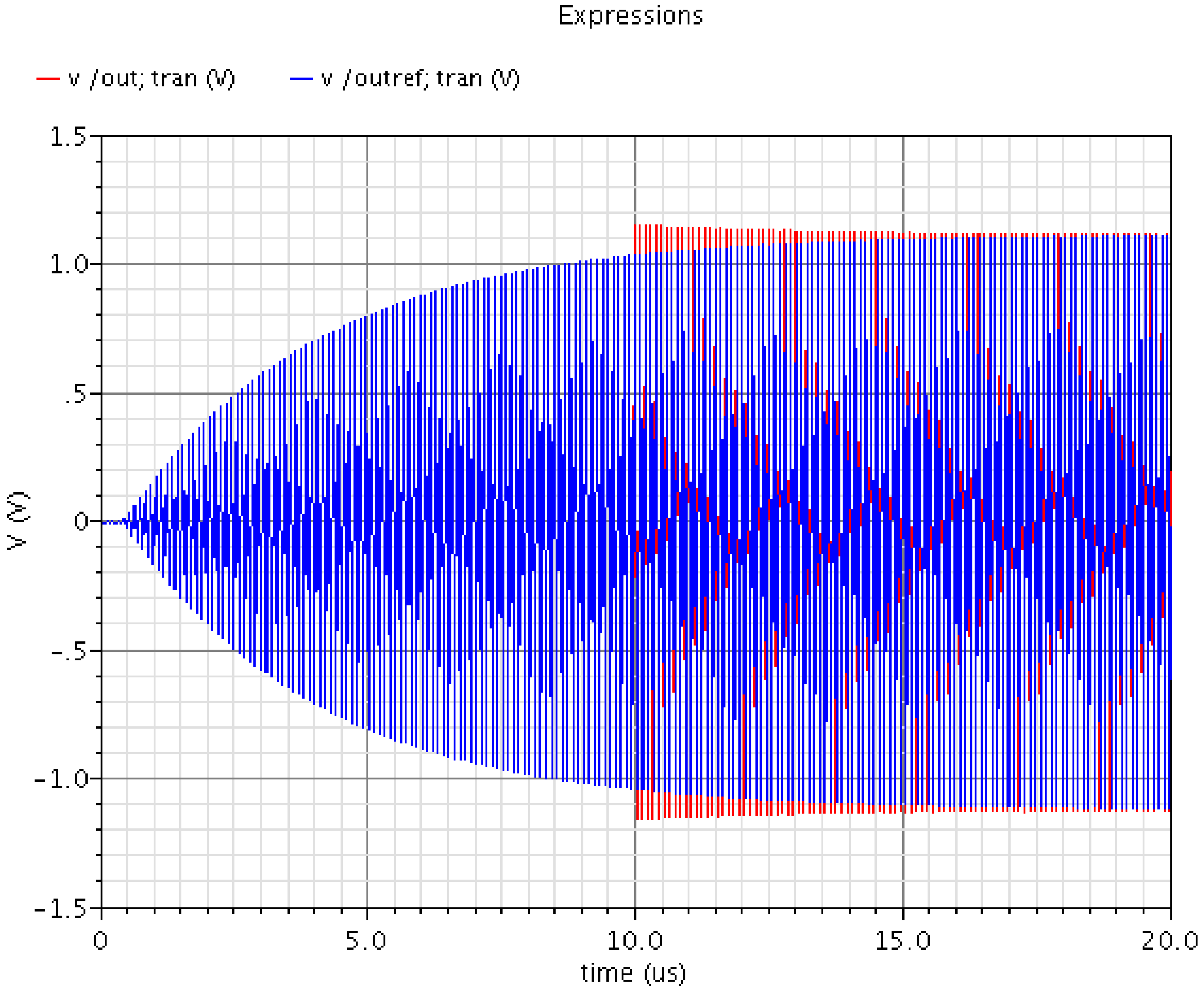
So, a driven circuit with loss, will only have a transient phase change, such that the ISF will not be periodic, and decay, typically, exponentially to zero. This means that the steady state ISF must go to zero, so that there is no steady state phase modulation, as is required using standard LTI theory.

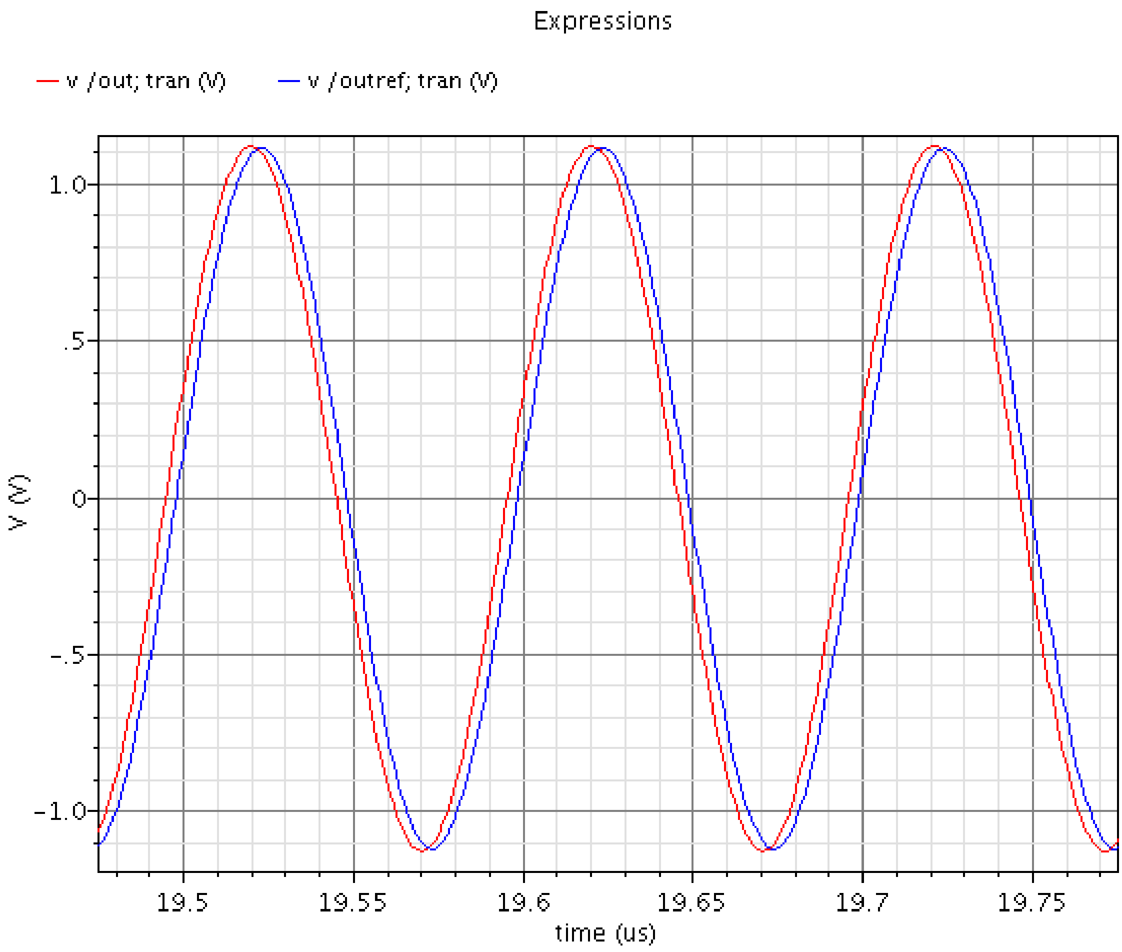
ISF Oscillator Schematic

Consider a simple tuned oscillator



ISF Oscillator waveforms





the above simulations results, show a persistent steady phase change arising from a single impulse, as required for the HL model.

However, despite the apparent nonzero steady state ISF, simulations have shown that without time constant variation with signal amplitude, up conversion generated phase noise is minimal. This implies that despite correction of oscillator losses, a real oscillator can behaves as if its ISF is very low, or zero for low frequencies.

The above schematic shows a phase shift of around 0.3 radians, for a pulse current of 500ua@10ns@20pf @1V, such that equation 15 of [2] becomes of the order of:

$$\phi(t) = 2\times 10^{-3}\,\frac{\sin(\Delta\omega t)}{\Delta\omega} \text{ per p}a$$

or -54 dBc for a 1pa @ 1hz noise signal.

Subsequent simulations with a huge 15na low frequency noise injection, showed that any up-conversion by this method would have to be < -50db, rather than the gross +30dB calculated above.

What went wrong? Why is there no up conversion?

From the point of view of the HL theory, this is explained by the notion that the up conversion gain constant is determined by the average value of the ISF waveform. So, in this particular case the assumption is that C_0 is zero.

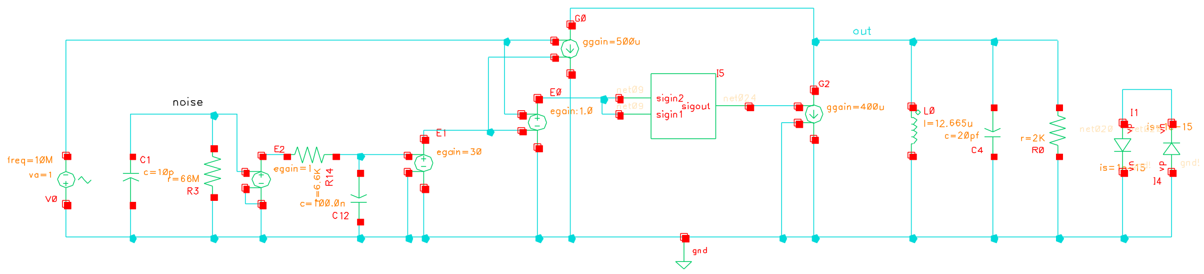
Linear/Non-Linear Phase Noise Simulation

Many different types of Cadence simulations were performed in order to confirm and gain a correct understanding of up converted phase noise, only some of which are included in this paper.

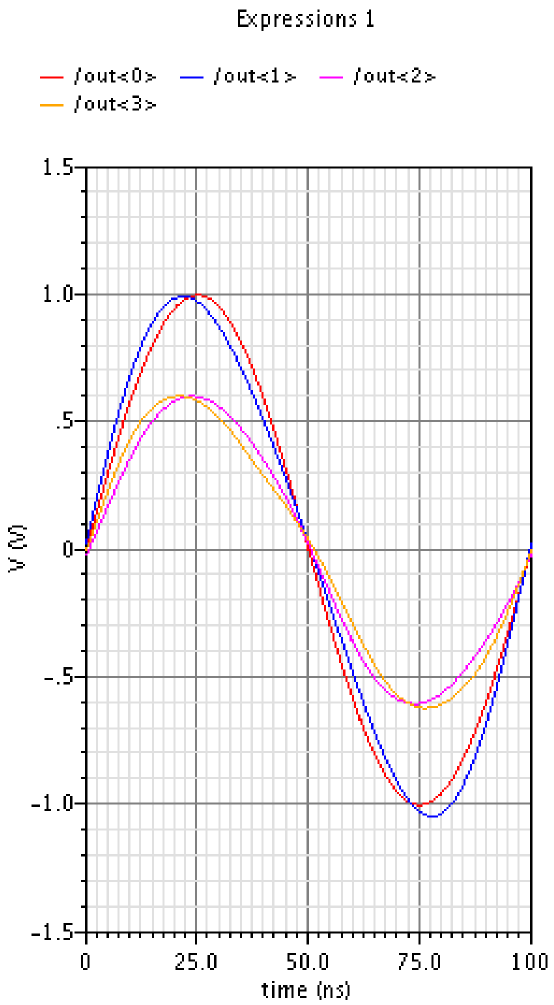
The following open loop schematic shows a system that was adjusted to produce linear and non-linear transfer functions. The system also allowed for the injection of additional, low frequency noise to verify mixed up-conversion effects.

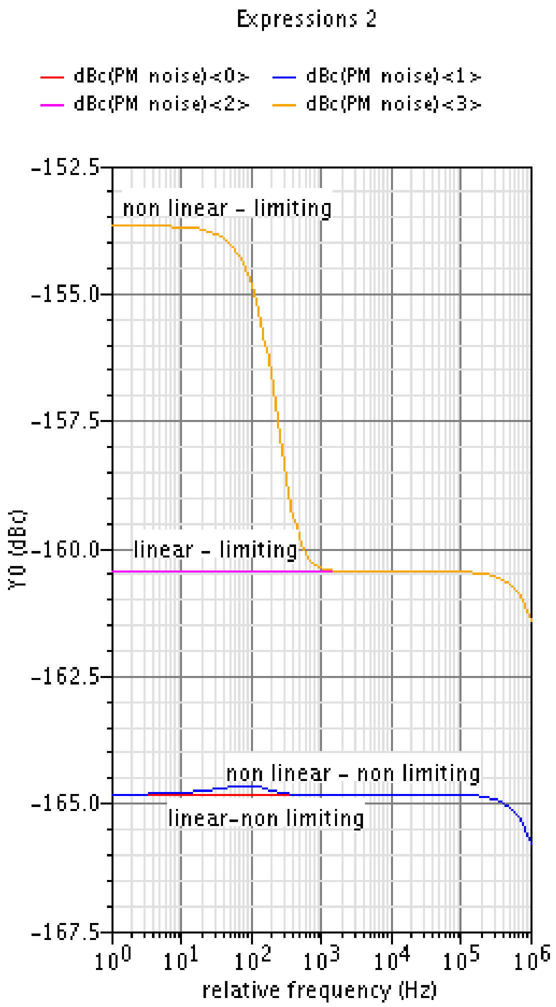
Cadence PSNoise analysis was then used to evaluate the conditions for up-conversion of noise.

Schematic



This schematic contains a square law component to generate square law non-linearities. Low frequency noise was 15na/rthz into the tank.

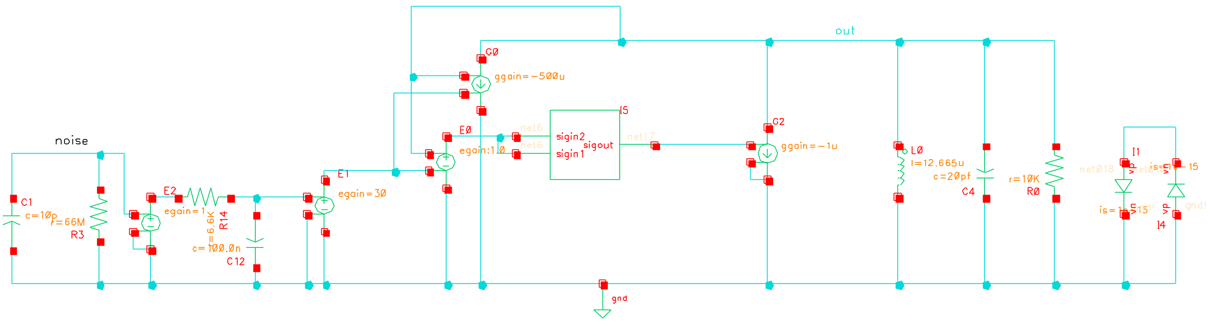




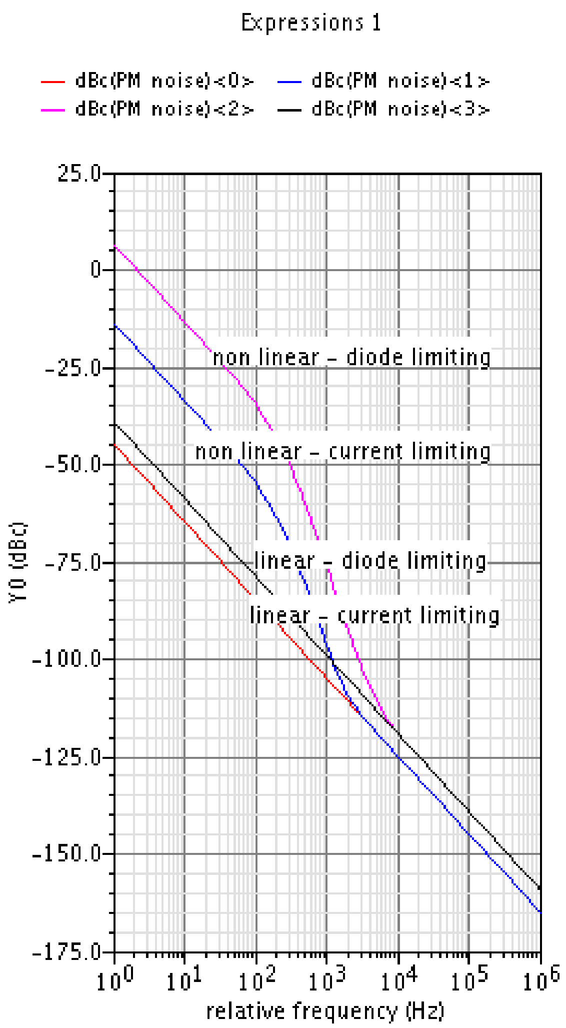
The above results show that without a non-linear procedure, up converted phase noise is negligible

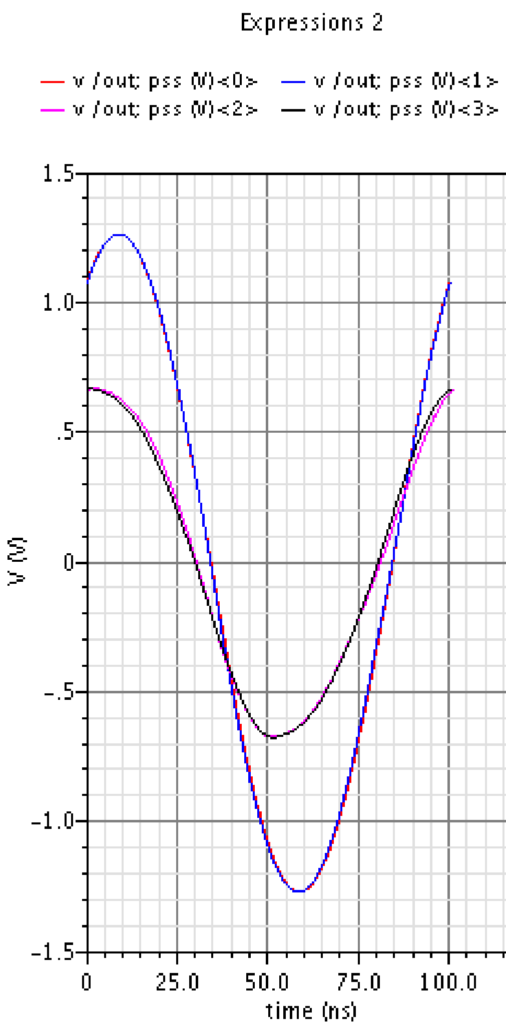
These results were confirmed with a similar configuration, but as an actual oscillator.

Oscillator Schematic



Oscillator Waveforms





Simulations showed that with minimal non-linearity (just enough to stabilise the oscillator), up- conversion of low frequency was essentially zero. Maximum up-conversion of noise occurred at maximum square law non-linearity, and heavy diode clamping.

In summary, the existence of baseband up converted noise frequencies in higher Q oscillators is not because of an assumption that an oscillator is a linear, but a time variant system with respect to a noise signal. Non-linearity is crucial in accounting for up-conversion.

The Correct Model

Consider an oscillator signal;

$$V_0(t) = V_p \sin(\omega_0(t) + \phi(t))$$

If the phase or frequency of this expression is a function of that signal, that is, the phase or frequency of the oscillator waveform is dependent on either the circuit's voltage or current at some point in the circuit, i.e. a non-linear circuit. e.g.

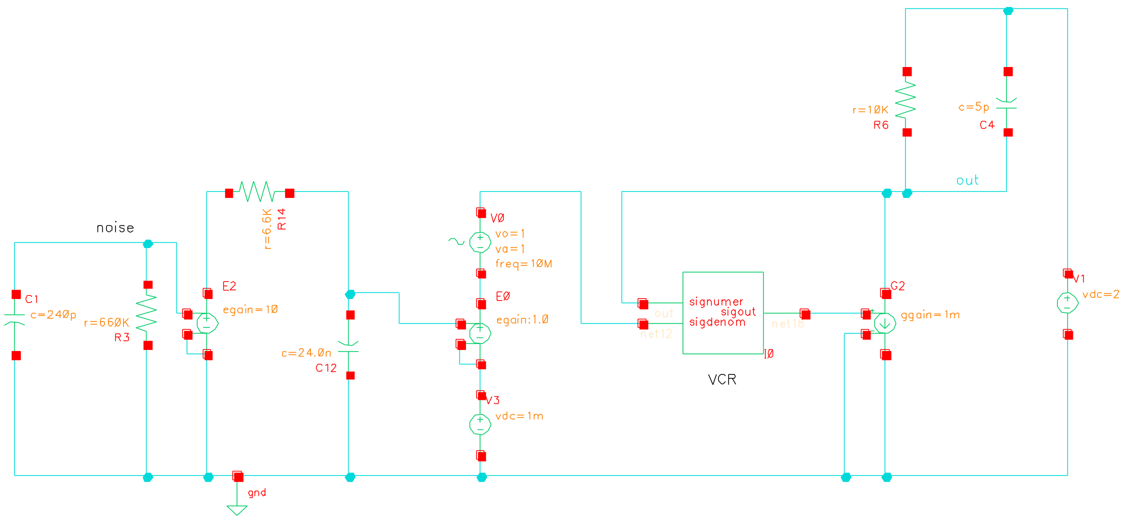
$$V_0(t) = V_p \sin(\omega_0(s_{mf}, t) + \phi(s_{mp}, t))$$

and for any real oscillator, this essentially, is always the case.

Then mixed products will be generated.

Consider an amplifier constructed using an ideal voltage controlled resistor:

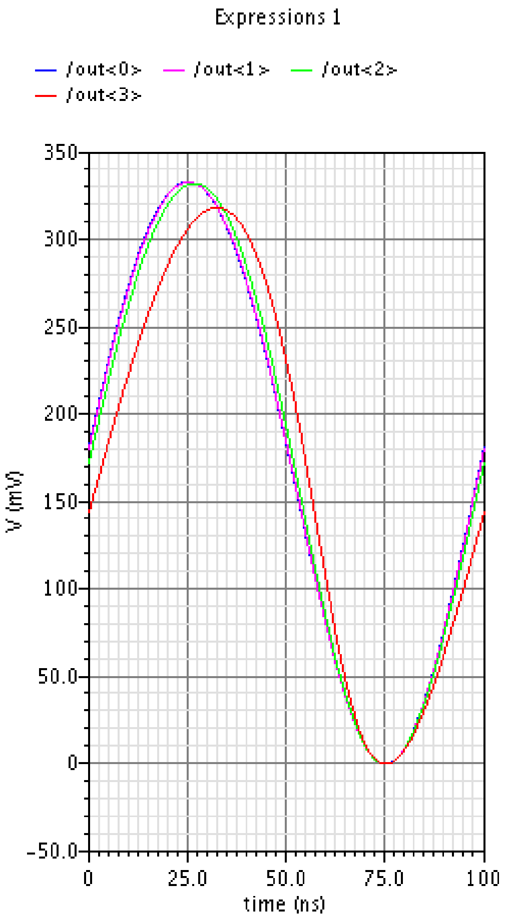
Amplifier with non-linear output resistance

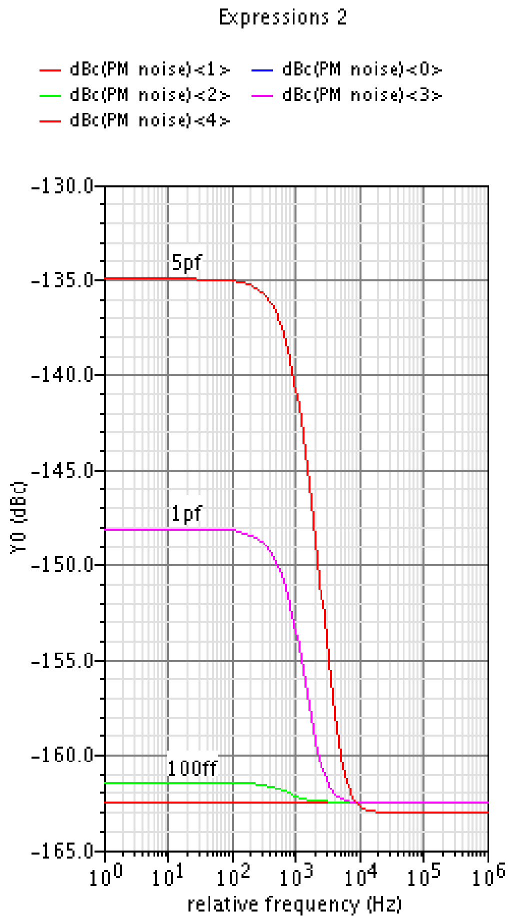


Note that this amplifier contains a fixed, linear capacitance. The block component is a divider used to construct an ideal linear VCR with a gm source. Additionally, there is a low frequency noise generator.

It is often claimed that simple non-linear mixed products do not generate phase noise, and sometimes the qualifier "memory less" is alluded to. It will now be shown that it is precisely this non-linearity, with memory, that is the root cause of up-conversion.

Signal and phase noise waveforms of non-linear output resistance amplifier.





These waveforms show a dramatic up-conversion of low frequency noise, dependant on the value of load capacitor.

In producing an output by varying a resister, the time constant of the output network changes with signal voltage. This means that signal phase shift is dependent on applied voltage, and hence phase shift is dependent on any applied low frequency noise voltages. Therefore a phase modulator has been produced. This results in up-conversion of low frequency noise.

Unfortunately, the literature abounds with the notion that non-linear amplifiers are modelled with Taylor series, when in reality, they need to be analysed with Volterra series for any condition but zero frequency.

Non-linear time constants can be produced in many ways. The most obvious is with non-linear capacitors, of which all active devices exhibit to significant extent, especially as most oscillators usually go from hard on to full cut-off. One main offender are diode clampers. Time constants at a clamp point vary significantly over the signal cycle.

References:

[1] A. Demir - “On the Validity of Orthogonally Decomposed Perturbations in Phase Noise Analysis” -

[2] Hanjmira and Lee – “A General Theory Of Phase Noise in Electrical Oscillators” – IEEE Journal of Solid State Circuits, vol 33, no 2, February 1998

[3] Hanjmira and Lee – “Oscillator Phase Noise – A Tutorial”

[4] E. Hegazi, J. Rael, A. Abidi – “The Designers Guide to High Purity Oscillators”
