Phase Noise
Basic Principles

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Overview

Many descriptions of phase noise may be somewhat confusing to those on first encounter of the concept. Terms such as Power Spectral Density (PSD) are used despite the notion that "phase" itself is a unit-less term. Phase is expressed in radians or degrees, which has no inherent "power" concept associated with it, so why refer it phase noise as a power?

Introduction

Phase noise is a characteristic term that expresses phase disturbances to a desired signal. Associated with phase noise is *time jitter* of a signal, or simply jitter. Although variations in phase or time of a signal are equivalent, depending on the system, one is usually a more appropriate parameter to describe the resulting errors that are produced in the system. The consequences of phase noise are *that additional frequencies* are present in the spectrum of the power signal. These extraneous frequencies signals, for example, could cause the velocity of a Doppler radar system to be calculated incorrectly, or result in a radio transmission channel interfering with an adjacent channel. The consequences of jitter are that signal digital data could be "clocked" at an incorrect time, resulting in false data bits.

To illustrate the basic concept of phase noise, consider a single frequency signal that is expressed as:

$$V_x(t) = V_0 \sin(\omega_0 t + \phi_{pm}(t)) \quad - 1$$

Where $\phi_{pm}(t)$ is a term that expresses that fact that there are disturbances to the phase and timing of a signal at a constant angular frequency $\omega_0 t$. The effect of this phase term for a pure sinusoidal noise source is shown as follows.

Let:

$$\phi_{pm}(t) = k\sin(\omega_{pm} t) \quad - 2$$

And substitute 2 into 1

$$V_x(t) = V_0 \sin(\omega_0 t + k\sin(\omega_{pm} t)) \quad - 3$$

This expression can be expanded by standard trigonometry relations and the result for $\phi_{pm}(t) << 1$ is:

$$V_x(t) = V_0[\sin(\omega_0 t)) + k\cos(\omega_0 t)\sin(\omega_{pm} t)] \quad - 4$$

Or, expanding again:

$$V_x(t) = V_0[\sin(\omega_0 t)) + \frac{k}{2}(\sin(\omega_0 t + \omega_{pm} t) - \sin(\omega_0 t - \omega_{pm} t))] \quad - 5$$

This expression shows that sinusoidal phase noise voltages produce additional frequencies about the signal frequency. These additional frequencies are called sideband frequencies.

Note that the multiplication term of the noise signal with the signal is in quadrature of the signal frequency. That is, the noise signal is multiplied by $\cos(\omega_0 t)$

The importance of this, is that if it is assumed that the frequency of the signal is related to its zero crossings, it can be determined from equation 4, that the zero crossings of $V_x(t)$ will be changed by $\phi_{pm}(t)$.This means that if $V_x(t)$ is passed through a zero crossing detector and/or limiter in order to determine the fundamental signal frequency of the signal, this frequency will be effected by $\phi_{pm}(t)$. That it, it is a frequency error that can not be easily eliminated.

From expression 5, it can also be determined that the ratio of the amplitude of the sideband frequencies to the signal frequency amplitude is

$$R = \frac{k}{2} \quad - 6$$

And that, due to the sinusoidal nature of the assumed noise, the peak phase error is

$$\phi_{pmpeak} = \frac{k}{2} \quad - 7$$

Or

$$R = \frac{V_0 k/2}{V_0} = \phi_{pmpeak} \quad - 8$$

Where R is a characterisation number expressing the ratio of the signal voltage (power) to sideband voltage (power). *Therefore*, *phase noise, in radians, can be equated to the ratio of signal power to noise power*. Often, the phase noise is referred to as a Power Spectral Density (PSD), but strictly speaking, phase is not a power, but its relative frequency spectrum is still given by measurements of sideband power to signal power. That is, the effect of a phase noise disturbance is that additional frequencies are generated.

Calculating phase noise near the signal frequency

Consider equation 1

$$V_x(t) = V_0 \sin(\omega_0 t + \phi_{pm}(t)) \quad - 1$$

Now suppose that a 90 degrees phase shifted signal, with no noise is available:

$$V_{r0}(t) = V_{r0}\cos(\omega_0 t) \quad - 9$$

Then a new signal can be constructed by multiplication:

$$V_{xr}(t) = \alpha V_0 \sin(\omega_0 t + \phi_{pm}(t)) V_r \cos(\omega_0 t) \quad - 10$$

Which expands to:

$$V_{xr}(t) = \frac{\alpha V_0 V_r}{2}[\sin(2\omega_0 t)\cos(\phi_{pm}(t)) + (1+\cos(2\omega_0 t))\sin(\phi_{pm}(t))] \quad - 11$$

If this signal is passed through a low pass filter such that the frequencies of $\phi_{pm}(t)$ are $< 2\omega_0 t$, and if $\phi_{pm}(t)$ is $<< 1$, equation 11 is reduced to:

$$V_{xr}(t) = \frac{\alpha V_0 V_r}{2}\phi_{pm}(t) \text{ - 12}$$

Or

$$\Delta\omega_{an} = \omega_0 - \omega_{an} \text{ - 13}$$

The summary of this result, is *that if a signal is multiplied by a noise free signal that is 90 degrees shifted, and low passes filtered what remains,* bar a gain constant, *is the phase spectrum of the phase noise*.

General Phase Noise Analysis

For reference, the following expression is usually used as a starting point in analysing the details of phase noise:

$$V_x(t) = V_0 \sin(w_0 t + \phi_{pm}(t)) \quad \text{- 14}$$

Where $V_{am}(t)$ is an amplitude modulation (AM) noise term and where $\phi_{pm}(t)$ is a phase modulation (PM) noise term.

Amplitude Modulation (AM) Noise

From equation 10, amplitude noise may be evaluated. This is useful as it more easily clarifies the difference between AM noise and PM noise. Ignoring the PM terms of the equation results in:

$$V_x(t) = V_0(1 + V_{am}(t))\sin(\omega_0 t) \text{ - 15}$$

Suppose that:

$$V_{am}(t) = kV_{amp}\sin(\omega_{am} t) \text{ - 16}$$

Then:

$$V_x(t) = V_0(1 + kV_{amp}\sin(\omega_{am} t))\sin(\omega_0 t) \text{ - 17}$$

Expanding this equation gives:

$$V_x(t) = V_0(\sin(\omega_0 t) + kV_{amp}\sin(\omega_{am} t)\sin(\omega_0 t)) \text{ - 18}$$

$$V_x(t) = V_0(\sin(\omega_0 t) + kV_{amp}\cos(\omega_0 t - \omega_{am} t) - \cos(\omega_0 t + \omega_{am} t)) \text{ - 19}$$

Again, also showing the presence of sideband frequencies. However, in contrast to the phase modulation case, examination of equation 13 shows that $V_{am}(t)$ does not change the zero crossings of $V_x(t)$ generated by the signal. This is because for any product function,

$$y(x) = u(x)v(x) \text{ -20}$$

If $u(x) = 0$, then $y(x) = 0$ irrespective of $v(x)$ - 21

This means that, for example, if $V_x(t)$ is passed through a zero crossing detector and/or limiter in order to determine the fundamental signal frequency, this frequency will be unaffected by $V_{am}(t)$.

Additive Noise (AN) Noise

Consider the case of adding noise to a single frequency signal, which can be referred to as additive noise, or AN:

$$V_x(t) = V_0 \sin(\omega_0 t) + V_{ap}(t) \text{ - 22}$$

For a sinusoidal noise source:

$$V_x(t) = V_0 \sin(\omega_0 t) + V_{ap} \sin(\omega_{an} t) \text{ - 23}$$

This equation can be restated, with reference to frequency offsets from the main signal, as:

$$V_x(t) = V_0 \sin(\omega_0 t) + V_{ap} \sin(\omega_0 t + \Delta\omega_{an} t) \text{ - 24}$$

where $\Delta\omega_{an} = \omega_0 - \omega_{an}$

Expanding 20 gives:

$$V_x(t) = V_0 \sin(\omega_0 t)(1 + \frac{V_{ap}}{2V_0} \cos(\Delta\omega_{an} t)) + V_0(\sin(\omega_0 t) + \frac{V_{ap}}{2V_0} \cos(\omega_0 t) \sin(\Delta\omega_{an} t)) \text{ - 25}$$

Which on comparison to the results for AM and PM, it can be seen that AN can be reformulated as equal parts of equivalent AM and PM noise.

Clippers, Clampers, Comparators and Limiters

A distinction will now be made being clampers, clippers, simple limiters, and comparators.

Typical the term “limiter” may refer to simple clipping/clamping of a signal or to high gain amplifiers/comparators which limit their output when input signals reach certain levels. To avoid confusion, clippers and clampers will be regarded as simple lopping of the tops and bottoms of the amplitudes of a signal, and limiters and comparators will be taken to be relatively high gain amplifiers that, essentially, form zero crossing detectors.

Clipper/Clampers

Consider passing a signal through a memory less clipper, where the clipping levels are sufficiently distant from the zero crossings of the signal. In this instance, the zero crossings of the signal are unaffected and hence, the output phase noise of the clipper system will not be changed. This is the often given example of eliminating AM noise in a PN sensitive system by “limiting” the amplitude variations.

Limiters/Comparators

Limiters/Comparators are a bit more difficult to analyse. One standard approach is to model the gm of the amplifier as a square wave and multiply the signal by the Fourier expansion of the square wave. However, the simplest method is to simply use a simulation program.

Cadence Spectre RF - Cadence Spectre RF is a software program that solves the complicated non-linear differential equations that occur when analysing phase noise. It is known to reliably calculate phase noise over a wide range of conditions.

Conventional wisdom e.g. C.Samori et al “spectrum folding and phase noise in LC tuned oscillators”, asserts that the signal to phase noise at the output to an ideal comparator is the same as its input signal to noise ratio. However, *this is only true when the input and output are band limited.* In reality, it is often extremely difficult to filter the signals such that that condition can be fulfilled, and for the internal noise of a transistor or resistor, it is simply impossible to filter.

Real High Gain Limiters/Comparators Are Intrinsically Noisy

Spectre RF simulations show that if a source has wideband noise, for example a simple sum of a signal and resister thermal noise, without input band limiting close to the signal frequency, the output phase noise around the carrier, will intrinsically be, some $12dB/\sqrt{Hz}$ worse. This is due to folding of harmonically mixed noise. This will always be so for internal noise of the input active devices of the comparator as they cannot be band limited. Internal device noise is thus effectively amplified by 12db. The practical result is that *a high gain limiter amplifier* in a well designed oscillator system used to "square up" a low noise oscillator *is the single most dominate source of noise degradation to the oscillator signal.* That is, in a well designed oscillator system, if the comparator is not the dominant noise source, there is probably something wrong in the design.

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