Analogue Design
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Feedback Stability

The only guaranteed way to make a small fortune, is to start with a large one, and lose some of it.

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Abstract

This paper forms an introduction to feedback theory as it applies to electronic amplifiers. The approach is that of engineering utility rather than a mathematical precise expose, as it is not possible to fully understand stability theory without an understanding of basic complex analysis. It should be noted that this paper assumes some basic understanding of frequency and phase response of amplifiers.

Overview

The fundamental purpose of feedback theory is to prevent an amplifier oscillating, or to ensure an oscillator will, to wit:

"Amplifiers do, oscillators don't", is the usual design problem.

There are various arguments that are used to give conditions for oscillations. Some of these are a bit dubious in the sense that they are not usually sufficient or necessary. The general problem of stability in quasi-linear systems was solved by Nyquist in the 1930's, or thereabouts. The argument is rather technical, but is necessary because a loose analysis of the issue results in a conclusion at odds with experiment.

A typical argument goes along the line of:

Suppose the gain is grater than unity, and that the net phase shift around the loop is 0 degrees. If a small signal exists at the input of an amplifier, it will become larger at the amplifiers output, and as its connected to its input will amplifier again etc, until the amplifier limits and forces a gain of exactly 1, such that this state will persist indefinitely. This is sometimes called the *Barkhausen Criteria*, and is false.

This argument is a bit daft really. By itself, it implies that the amplifier will latch hard to one rail forever!

Even if this argument is extended in more detail it still leads to a prediction that disagrees with experiment namely:

An amplifier can have net positive feedback, with gain greater than one, and yet still be stable.

The reason that attention is drawn to this statement is that it is not unknown for individuals to have formal courses in stability theory, correctly make predictions about transfer functions, yet are quite unaware of what the physical implication of those transfer functions are. Indeed, even professors discuss such "conditionally stable systems" in detail, quite oblivious to the fact that it means that such systems are positive feedback systems with gain greater than one.

Note:

A negative feedback signal is equivalent to a 180 phase lag. This means that an additional 180 degrees of phase lag will generate a total of 360 degrees, or a net 0 degrees phase shift around the loop.

Stability Criteria

The understanding of stability criteria requires complex analysis, so the results will be stated for reference but in practice they can be expressed in simpler terms, which will be addressed below.

In general an amplifier will have a *small signal loop gain* transfer function given by:

$$LG(s) = A_0 \frac{(1+s\tau_a)(1+s\tau_b)1+s\tau_c)...}{(1+s\tau_1)(1+s\tau_2)1+s\tau_3)...}.B(s)$$

That is, an amplifier will have a number of poles and zeros, or alternatively a number of gain roll off and gain increases. B(s) is also of the same form, and is typically the feedback network. Note that it is the gain around the loop, not just the amplifier gain. The loop gain is thus the open loop gain multiplied by the feedback divisor.

If there is a pole in the right half plane of LG +1, it can be shown that the impulse response of any transfer function described by LG+1, will result in an *ever increasing sinusoidal response*. In practice this increase will limit when the amplifier saturates at its supply rails. This results in steady state oscillations. This implies that the amplifier is analyzed such that it is reasonable linear for small signals, but can limit at large signals in a way such that this limiting does not change the small signal phase response at the limiting point.

Nyquist Theorem

The basic issue with the above transfer function is that it is usually obtained as a ratio of polynomials, not in factored form from which the poles and zeros can be determined. Nyquist's Theorem is an approach that determines *whether or not there is a pole in the right half plane* by a *graphical technique*.

As far as *general amplifier design is concerned,* Nyquist's Theorem, in the authors opinion, *has no value whatsoever*. It is described here only for reference in a historical context. Modern and cheap computer simulation programs have completely eliminated any requirement for drawing "Nyquist Plots", even on the computer. There may be some value in control systems analysis, but even in this case, there are fundamental issues that still require stability to be addressed by other methods.

For those interested in an exact statement of Nyquist's Theorem, a search on www.google.com will turn up rather a lot.

A key issue in stability design is that a real system is nonlinear, such that a small signal analysis that suggests stability is often incorrect. Typically, when an amplifier goes into limiting, the phase response is much worse than that indicated at the small signal bias conditions. Indeed, the only practical way to determine stability accurately is to run transient simulations, with the ac response being only used as a basic initial design guide.

Stability and Bode Plots

There is some debate on exactly what a "Bode" plot is. To wit, is it just a generic gain against frequency plot, or is it the straight-line approximation to the exact gain against frequency. For the purpose of stability analysis, the general technique is to discuss in terms of the straight-line approximations.

Arguable, stability in 99.99% of cases, in general amplifier design, can be summed up as follows:

A - *If the slope of the loop gain is -20db/dec when the loop gain finally falls to 0db, the system will be stable, and unstable if the slope is greater than -20db.*

Prior to simulation tools, checking for this condition was a reasonable method. With the advent of simulation tools, the phase is checked directly, which results in the following:

B - *If the phase of the loop gain is less than 180 degrees when the loop gain finally falls to 0db, the system will be stable, and unstable if the phase is greater than 180 degrees.*

This is the probably the most useful concept with which to tackle the stability problem. The general procedure is to set up a circuit simulation that plots the gain and phase of the loop gain. The response is then checked to ensure the above condition is satisfied. Note the use of the word "final", it is the phase shift at the final 0db-gain point that determines stability. It is immaterial if the phase goes to 180 degrees prior to the 0db point so long as it gets to less than 180 degrees before the final unity gain point.

There is one provider to the above. If the phase does go to 180 degrees then recovers, this is usually a system known as conditionally stable. The issue here is that if the loop gain falls due to component tolerances, power supply ramp ups etc, it is possible for the gain to become 0db at a frequency such that the phase has not recovered, and hence the amplifier could become unstable.

For reference, sometimes A is equivalently expressed as:

If the intersection of open loop gain and closed loop gain is at a rate of -20db/dec, the system will be stable, and unstable if greater than -20db

Because the loop gain is the difference between open loop gain and closed loop gain when expressed in db's. In this procedure, both the open loop gain and closed loop gain are plotted together, and their intersection examined.

So, a gain roll off of 40db/dec will be unstable, noting that minus means a decrease in gain, and "greater" implying a larger *negative* roll off, as usually understood, but possible a bit misleading. One should use a bit of logic in the following as to whether there is an implied negative to the gain response rates.

Statement (A) and (B) implies that if a higher order roll off of say 80db/dec, is then "compensated" such that when the gain goes through the 0db point it is only at a 20db slope, then the system will be stable. This is despite the fact that in the 80db/dec region the phase may approach n*90 degrees, which would have violated the naive interpretation of a stability requirement.

Technical Points

Technically, a negative feedback system with only a pure second order roll off can not be unstable, as a second order system can only approach 180 degrees phase shift at infinity. However, it is understood that *any practical amplifier will always have at least few degrees of excess phase shift that will always result in the system oscillating.*

Basic Compensation

The key concept in stability compensation is that each 20db/dec roll off creates a phase shift that tends to generate additional 90 degrees lags. The idea is to somehow generate loop gain boosts that increases the phase from excessive negative values at the 0db loop gain point.

General Concepts

The loop gain frequency response will have terms that result in:

1 Gain roll offs (poles) of n.20db/dec or phase shifts tending to -90 degrees, often called a lag.

2 Gain increases (zeros) of m.20db/dec, or phase shifts tending to +90 degrees often called a lead.

2 Flat gain regions.

It is also possible to get lag phase shifts with flat gain response. These are called "all pass" networks, and generally sound a death knell in a feedback amp.

For example, consider a (ladder) set of series resistors (A), with their junctions connected to a series resistor and capacitor to ground (B)., with an input and output at the resistor string ends. If the resistors in series with the capacitors are much smaller than the (A) resistors then each of the (A) resistors and its capacitors will generate a 20db/dec roll off. At a sufficiently high frequency the (B) resistors and capacitors will generate a 20db/dec increase, such that at the highest frequencies the response will be attenuated but flat.

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For general amplifiers, a basic design principle is that there is no extra gain available at high frequencies. This means that if you want to turn a 40db/dec slope into a 20db/dec slope, you have to through away some gain at lower frequencies and recover it back again at high frequencies. This can be done in a number of ways

1 Place a series RC across the input of the amplifier.

2 Place a series RC across an internal gain setting node of the amplifier.

These techniques all reduce the loop gain prematurely because of the capacitor, but at high frequencies the series resistor limits the shunting effect of the capacitor, thereby removing the phase shift introduced by the capacitor. A key idea here, is that reducing the gain earlier in frequency means that additional poles in the amplifier will (hopefully) only occur well after the 0db point, hence will not affect the amplifiers stability.

Another technique, but basically the same principle in disguise, is to:

3 Bypass a feedback resistor with a capacitor.

This increases the loop gain because the feedback resistors had already attenuated the gain.

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