Analog Design
Kevin Aylward B.Sc
Distortion Reduction
By The Addition of High Frequency Bias
In Analogue Tape Recorders

[Back to Contents](#)

Abstract

This paper derives the mathematical basis of one aspect as to why adding high frequency bias to an audio signal increases the signal to distortion ratio. That is, distortion is effectively reduced by the addition of a high frequency bias signal. Hysteresis is not covered.

The literature is often quite vague on this matter, often giving explanations more in line with explanations concerning the dc bias used in an AC transistor amplifier. These explanations are generally quite erroneous and more often then not, say very little about anything. It is shown here, that the fundamental principle involved in adding H.F. bias is more akin to the well-known technique of parametric amplification. That is, the technique where a H.F. pump signal is added to an information carrying signal across a non-linear capacitor, thereby effecting almost noiseless amplification of the information signal.

Analysis

By inherent assumption, it is assumed that the signal decoded from the tape has distortion. This output signal can therefore be expressed by:

$$V_o = a_0 + a_1 V_i + a_2 V_i^2 + a_3 V_i^3 + a_4 V_i^4 + + \ldots \qquad -1$$

An assumption is now made that the linearity of the tape is symmetrical for negative and positive flux signals. This implies that there are no even terms in the above expansion for the output signal. In addition, any dc component is also absent. It will also be assumed that terms above degree 3 contribute little to the overall distortion. Equation (1) thus becomes.

$$V_o = a_1 V_i + a_3 V_i^3 \qquad -2$$

First consider that V_i is a sinusoidal signal:

$$V_i = V_s \sin(\omega_s t) \qquad -3$$

And substitute (3) into (2)

$$V_o = a_1 V_s \sin(\omega_s t) + a_3 V_s^3 \sin^3(\omega_s t) \qquad -3$$

Elementary trigonometry gives the following relation:

$$\sin^3(x) = \frac{1}{4}(3\sin(x) - \sin(3x)) \qquad -4$$

Using 4 in 3 gives:

$$V_o = a_1 V_s \sin(\omega_s t) + \frac{a_3 V_s^3}{4}(3\sin(\omega_s t) - \sin(3\omega_s t))$$

$$V_o = (a_1V_s + \frac{3a_3V_s^{\,3}}{4})\sin(\omega_s t) - \frac{a_3V_s^{\,3}}{4}\sin(3\omega_s t)$$

Whence in can be seen that the 3rd harmonic distortion ratio DR_3 is:

$$DR_3 = \frac{\frac{a_3V_s^{\,3}}{4}}{a_1V_s + \frac{3a_3V_s^{\,3}}{4}} \cong \frac{\frac{a_3V_s^{\,3}}{4}}{a_1V_s} = \frac{a_3V_s^{\,3}}{4a_1V_s} = \frac{a_3V_s^{\,2}}{4a_1} \qquad -5$$

as V_s and a_3 are assumed small.

Now consider the case when a h.f. bias signal is added to the audio signal:

$$V_i = V_s\sin(\omega_s t) \;+ V_b\sin(\omega_b t) \quad -6$$

and substitute equation (6) into equation (2)

$$V_o = a_1V_i + a_3V_i^3 \qquad -2$$

$$V_o = a_1(V_s\sin(\omega_s t) \;+ V_b\sin(\omega_b t)\;) + a_3(V_s\sin(\omega_s t) \;+ V_b\sin(\omega_b t)\;)^3$$

It is noted that the h.f. components are filtered from the final recovered signal, so the above can be immediately simplified to:

$$V_o = a_1V_s\sin(\omega_s t) \;+ a_3(V_s\sin(\omega_s t) \;+ V_b\sin(\omega_b t)\;)^3$$

Expanding:

$$V_o = a_1V_s\sin(\omega_s t) \;+ a_3(V_s^{\,3}\sin^3(\omega_s t) + 3V_s^{\,2}\sin^2(\omega_s t)V_b\sin(\omega_b t) + 3V_s\sin(\omega_s t)V_b^{\,2}\sin^2(\omega_b t)$$

The last term clearly generates harmonics at the bias f or 3f, so will be removed by the inherent audio bandwidth filtering. The second last term is a x cross modulation term, which is required to be retained. The 3rd last term can be removed so long as the modulation term from the (Fbias - 2F) signal is greater the audio bandwidth, i.e. F bias must be at least 3f signal, from:

3rd last term:

$$\sin^2(\omega_s t)\sin(\omega_b t) = \frac{1}{2}(1 - \sin(2\omega_s t))\sin(\omega_b t)$$

There is a component at the bias frequency, and components modulated about the bias frequency (from (2sin(a)*sin(b)=sin(a+b)-sin(a-b)).

The terms remaining are therefore:

$$V_o = a_1V_s\sin(\omega_s t) \;+ a_3(V_s^{\,3}\sin^3(\omega_s t) + 3V_sV_b^{\,2}\sin(\omega_s t)\sin^2(\omega_b t)\;) \qquad -8$$

Expanding the last term in (8)

$$\sin(\omega_s t)\sin^2(\omega_b t) \;\; = \sin(\omega_s t)\frac{1}{2}(1 - \cos(2\omega_b t))$$

Again, the modulation at 2 F bias is eliminated by the audio filtering, leaving only an additional component at the audio signal frequency. Therefore (8) becomes:

$$V_o = a_1 V_s \sin(\omega_s t) \ + a_3 (V_s^{\ 3} \sin^3(\omega_s t) + \frac{3}{2} V_s V_b^{\ 2} \sin(\omega_s t)\ ) \qquad \text{-9}$$

or

$$V_o = V_s (a_1 + \frac{3a_3}{2} V_b^{\ 2}) \sin(\omega_s t) \ + a_3 V_s^{\ 3} \sin^3(\omega_s t) \qquad \text{-9}$$

Where it can be now seen that the audio signal has been "amplified" by the bias signal, and that the main 3rd harmonic distortion remains the same as in the case with no h.f. bias added. Thus the signal to distortion ratio has been increased.

The effective reduction in distortion can now be evaluated from:

In (5)

$$DR_3 = \frac{a_3 V_s^{\ 2}}{4a_1} \qquad \text{-5}$$

From inspection, the new signal to distortion ratio is:

$$DR_{3b} = \frac{a_3 V_s^{\ 2}}{4(a_1 + \frac{3a_3}{2} V_b^{\ 2})} \qquad \text{-10}$$

However, the h.f. bias is typically very much larger then the signal, so (10) becomes:

$$DR_{3b} = \frac{V_s^{\ 2}}{6V_b^{\ 2}} \qquad \text{-11}$$

So that comparing the relative distortion with and without h.f. bias:

$$\frac{DR_{3b}}{DR_3} = \frac{\frac{V_s^{\ 2}}{6V_b^{\ 2}}}{\frac{a_3 V_s^{\ 2}}{4a_1}} = \frac{2a_1}{3a_3 V_b^{\ 2}}$$

Conclusion

It has been shown that h.f. bias reduces signals by "parametric" amplification of the audio signal with respect to the 3rd harmonic distortion. It has also been shown that the h.f. bias signal must be at least 3 times the highest audio frequency.

It should be noted that commercial adjustment of bias is often performed using a 3rd harmonic distortion analyzer and setting the bias large enough that distortion is reduced, but not so large that audio h.f is response is effected significantly.
