Einstein's Mass-Energy Equation

From

Conservation of Energy and Conservation of Momentum

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Overview:

There are many alternate theories proposed to Einstein's famous $E = mc^2$. This paper gives those Tellytubbian views a right good shafting. Without naming any typical offenders (David de Hilster, http://www.autodynamics.org) some, are fatally flawed. This is due to the allowance of infinitely continuous pints of Guinness, or none at all. With that in mind, which is somewhat difficult after a few, the following is a derivation of the only form of the mass equation possible, assuming energy and momentum conservation are both still valid, which I do as my mum said so.

Proof:

So, lets start with the well-known old stuff by the man who stood on the jolly green giant, to wit the relations of energy and momentum, thus:

$F = \frac{d(mv)}{dt}$ Conservation of momentum - (1)

$KE = \int Fdl$ Conservation of energy - (2)

If we assume that the kinetic energy of a moving body is determined by any arbitrary function of velocity, and simply allow the ~~Guinness~~ mass*(see bottom of page) to vary with velocity we have, in very general terms:

$KE = k^2 m(v) + \alpha$ - (3)

Where m(v), the mass, is an arbitrary function of velocity, k^2 and α are arbitrary constants.

We can therefore immediately write:

$$k^2 m(v) + \alpha = \int \frac{d(mv)}{dt} dl$$

And since, $\frac{dl}{dt} = v$, we can write

$$k^2 m(v) + \alpha = \int v \frac{d(mv)}{dt} dt$$

Differentiating both sides of the equation w.r.t:

$$k^2 m' = v\frac{d(mv)}{dt}, where\, m' = \frac{dm}{dt}$$

$$k^2 m' = v^2 m' + vv' m, where\, v' = \frac{dv}{dt}$$

$$=> k^2 m'(1 - \frac{v^2}{k^2}) = vv' m$$

$$=> \frac{m'}{m} = \frac{vv'}{k^2(1 - \frac{v^2}{k^2})}$$

This equation can be immediately integrated as the R.H.S. is an exact differential:

$$Ln(\beta m) = -\frac{1}{2} Ln(1 - \frac{v^2}{k^2})$$

$$\therefore\ m = \frac{m_o}{\sqrt{(1 - \frac{v^2}{k^2})}} \text{ - (4)}$$

So there you have it, no sweat at all. The form of the classic relativistic mass equation, without all that high-brow space-time posturing taken by some of those people that think they're clever, but are really just a pain in the arse to those of us that are.

Although it is not possible to show that k is c, the speed of light, in this derivation, we know its so cos Einstein said so, so it must be true.

And substituting back into to - (3), with suitable initial conditions will give:

$$KE = k^2(m - m_o)$$

Or $KE = k^2 \Delta m$

And, cos we need this later for the "GR for Teletubbys" section we rearrange, with k=c:

$$mc^2 = KE + m_0 c^2$$

and with

$$\gamma = \frac{1}{\sqrt{(1 - \frac{v^2}{k^2})}}$$

then:

$$m_0 \gamma c^2 = KE + m_0 c^2$$

Now *define* the total energy, E by:

$$E_t = m_0 \gamma c^2$$

So that

$$E_t = KE + m_0 c^2$$

Identifying a rest energy term and a kinetic energy term. This bit is all rather a bit waffley really, but there you go…

And as a note, this concept of mass variation is really a little dated. Modern SR does not consider the mass to vary. Mass is an invariant. Momentum is redefined to achieve the same, but much more general effect as the result shown here.

Summary:

From these equations, all other relativistic equations can be derived, such as length contraction and time dilation. This direct derivation of the Einstein equation makes alternative theories quite dubious, in as much that the most fundamental, basic laws of Physics would have to be replaced, i.e. the laws of Energy and/or Momentum conservation.

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http://www.kevinaylward.co.uk/gr/index.html

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