General Relativity For Tellytubbys

Geodesic Deviation and Potbellied Mr. Riemann

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Overview

This section attempts to give a handle on the Riemann curvature tensor. I have had a bit of bother with this one as I could not find a really decent web site to steal the derivations from. They all missed out the bits, which I consider are crucial. For example, *the* fundamental point of the Riemann tensor, as far as G.R. is concerned, is that it describes the *acceleration* of geodesics with respect to one another. Some sites noted this fact, but did not show in their derivations how that particular derivation *actually* related to this acceleration. Taking vectors on round trips with talks of parallel transportation don't immediately explain what's happening, although very impressive sounding it is, indeed. Of course it's probably that I'm just too thick to see it. In addition, of course, all derivations left most of the details to one's futile imagination. I am led to believe that many people don't have a bleeding clue what's going on, although they can apply the formulas in a sleepwalking sense.

Further point. It is what are called, *tidal* forces that are equivalent to the acceleration of geodesics (geodesic deviation). If you consider the Newtonian, inverse square force law, at different radiuses, there is an effective differential force that tries to pull apart objects.

Consider Tinky-Winky and Dipsy orbiting the earth with some velocity, in what are assumed to be geodesics. Since they are not the same geodesics, Tinky-Winky and Dipsy may or may not move closer or further away from each other. The Riemann curvature tensor is what tells one what that acceleration between the Tellytubbys will be. This is expressed by

$$\mathbf{a}_w = \frac{D^2 w^a}{D\lambda^2} = R^a{}_{bcd} \frac{dx^b}{d\lambda} \frac{dw^c}{d\lambda} \frac{dx^d}{d\lambda}$$

or, equivalently

$$\mathbf{a}_w = \frac{D^2 w^a}{D\lambda^2} = R^a{}_{bcd} v^b w^c v^d$$

or equivalently, in posher notation

$$\mathbf{a} = \nabla_{\mathrm{v}} \nabla_{\mathrm{v}}\, \mathrm{w} = R^a{}_{bcd} v^b w^c v^d$$

where D is the covariant derivative operator, **w** is the separation vector between the Tellytubbys geodesic, and **V** is the parameterized velocity of the Tellytubbys as they travel on their geodesics. The last form is the second covariant derivative of the connecting vector w in the direction of v, the gist of this will be shown

Calculation of Riemann

This section calculates what the Riemann tensor is, it is then shown afterwards how this is related to the concept of acceleration described above.

First, lets note some prior results,

$$T^{\alpha}{}_{;\beta} = T^{\alpha}{}_{,\beta} + \Gamma^{\alpha}{}_{\mu\beta} T^{\mu}$$

$$T^{\alpha}{}_{\beta;\rho} = T^{\alpha}{}_{\beta,\rho} + \Gamma^{\alpha}{}_{\mu\rho} T^{\mu}{}_{\beta} - \Gamma^{\mu}{}_{\beta\rho} T^{\alpha}{}_{\mu}$$

For a normal second order partial derivative, we have

$$\frac{\partial^2 \Phi}{\partial x \partial y} = \frac{\partial^2 \Phi}{\partial y \partial x}$$

For the covariant derivative of a vector this is not true in general. i.e.

$$V_{;\alpha\beta} \neq V_{;\beta\alpha}$$

So, lets calculate what the difference on a vector **A** is

$$[\nabla_{\alpha}, \nabla_{\beta}] = \nabla_{\alpha}\nabla_{\beta} - \nabla_{\beta}\nabla_{\alpha} = V_{;\alpha\beta} - V_{;\beta\alpha}$$

oh, and the first term above is called a commentator, and this does get rather messy, but there you go that's G.R. for you, and I dropped the A on the LHS just to keep things unclutted.

$$(A^{\mu}{}_{;\alpha})_{;\beta} = (A^{\mu}{}_{,\alpha} + \Gamma^{\mu}{}_{\alpha\zeta} A^{\zeta})_{;\beta}$$

$$\begin{aligned}(A^{\mu}{}_{;\alpha})_{;\beta} = {} & (A^{\mu}{}_{,\alpha} + \Gamma^{\mu}{}_{\alpha\zeta} A^{\zeta})_{,\beta} \\ & + \Gamma^{\mu}{}_{\beta\gamma}(A^{\gamma}{}_{,\alpha} + \Gamma^{\gamma}{}_{\alpha\zeta} A^{\zeta}) \\ & - \Gamma^{\gamma}{}_{\beta\alpha}(A^{\mu}{}_{,\gamma} + \Gamma^{\mu}{}_{\gamma\zeta} A^{\zeta})\end{aligned}$$

You might have to think a bit about the above, but its just treat the first derivative as one big 2nd rank tensor, contravariant one, covariant one sort of thing.

$$(A^{\mu}{}_{;\alpha})_{;\beta} = (A^{\mu}{}_{,\alpha} + \Gamma^{\mu}{}_{\alpha\zeta} A^{\zeta})_{,\beta} + \Gamma^{\mu}{}_{\beta\gamma} A^{\gamma}{}_{;\alpha} - \Gamma^{\gamma}{}_{\beta\alpha} A^{\mu}{}_{;\gamma}$$

That one above, I thought quite neat when I first worked it out. Once again, see what dummy index's are swapped here

$$(A^{\mu}{}_{;\alpha})_{;\beta} = (A^{\mu}{}_{,\alpha} + \Gamma^{\mu}{}_{\alpha\zeta} A^{\zeta})_{,\beta} + \Gamma^{\mu}{}_{\beta\zeta} A^{\zeta}{}_{;\alpha} - \Gamma^{\zeta}{}_{\beta\alpha} A^{\mu}{}_{;\zeta}$$

$$(A^{\mu}{}_{;\alpha})_{;\beta} = A^{\mu}{}_{,\alpha\beta} + \Gamma^{\mu}{}_{\alpha\zeta,\beta} A^{\zeta} + \Gamma^{\mu}{}_{\alpha\zeta} A^{\zeta}{}_{,\beta} + \Gamma^{\mu}{}_{\beta\zeta} A^{\zeta}{}_{;\alpha} - \Gamma^{\zeta}{}_{\beta\alpha} A^{\mu}{}_{;\zeta}$$

Now swap all the alphas and betas, but note that the Christoffel symbols are symmetric, so we can swap those ones back again.

$$(A^{\mu}{}_{;\beta})_{;\alpha} = A^{\mu}{}_{,\beta\alpha} + \Gamma^{\mu}{}_{\beta\zeta,\alpha} A^{\zeta} + \Gamma^{\mu}{}_{\beta\zeta} A^{\zeta}{}_{,\alpha} + \Gamma^{\mu}{}_{\alpha\zeta} A^{\zeta}{}_{;\beta} - \Gamma^{\zeta}{}_{\beta\alpha} A^{\mu}{}_{;\zeta}$$

Now to subtract and collect terms, note the first and last term obviously cancels

$$V_{;\alpha\beta} - V_{;\beta\alpha} = (\Gamma^{\mu}{}_{\alpha\zeta,\beta} - \Gamma^{\mu}{}_{\beta\zeta,\alpha})A^{\zeta}$$
$$+ -\Gamma^{\mu}{}_{\alpha\zeta}(A^{\zeta}{}_{;\beta} - A^{\zeta}{}_{,\beta})$$
$$- -\Gamma^{\mu}{}_{\beta\zeta}(A^{\zeta}{}_{;\alpha} - A^{\zeta}{}_{,\alpha})$$

$$V_{;\alpha\beta} - V_{;\beta\alpha} = (\Gamma^{\mu}{}_{\alpha\zeta,\beta} - \Gamma^{\mu}{}_{\beta\zeta,\alpha})A^{\zeta}$$
$$- \Gamma^{\mu}{}_{\alpha\zeta}\Gamma^{\zeta}{}_{\rho\beta}A^{\rho}$$
$$+ \Gamma^{\mu}{}_{\beta\zeta}\Gamma^{\zeta}{}_{\rho\alpha}A^{\rho}$$

And with more index swapping

$$V_{;\alpha\beta} - V_{;\beta\alpha} = (\Gamma^{\mu}{}_{\alpha\zeta,\beta} - \Gamma^{\mu}{}_{\beta\zeta,\alpha})A^{\zeta} - \Gamma^{\mu}{}_{\alpha\rho}\Gamma^{\rho}{}_{\zeta\beta}A^{\zeta} + \Gamma^{\mu}{}_{\beta\rho}\Gamma^{\rho}{}_{\zeta\alpha}A^{\zeta}$$

$$V_{;\alpha\beta} - V_{;\beta\alpha} = (\Gamma^{\mu}{}_{\alpha\zeta,\beta} - \Gamma^{\mu}{}_{\beta\zeta,\alpha} - \Gamma^{\mu}{}_{\alpha\rho}\Gamma^{\rho}{}_{\zeta\beta} + \Gamma^{\mu}{}_{\beta\rho}\Gamma^{\rho}{}_{\zeta\alpha})A^{\zeta}$$

So, now the bit in brackets is, as you might have guessed is the Riemann tensor

$$[\nabla_{\alpha}, \nabla_{\beta}] = \Gamma^{\mu}{}_{\alpha\zeta,\beta} - \Gamma^{\mu}{}_{\beta\zeta,\alpha} - \Gamma^{\mu}{}_{\alpha\rho}\Gamma^{\rho}{}_{\zeta\beta} + \Gamma^{\mu}{}_{\beta\rho}\Gamma^{\rho}{}_{\zeta\alpha}$$

$$[\nabla_{\alpha}, \nabla_{\beta}] = \Gamma^{\mu}{}_{\zeta\alpha,\beta} - \Gamma^{\mu}{}_{\zeta\beta,\alpha} - \Gamma^{\mu}{}_{\alpha\rho}\Gamma^{\rho}{}_{\zeta\beta} + \Gamma^{\mu}{}_{\beta\rho}\Gamma^{\rho}{}_{\zeta\alpha}$$

and just to make it agree with the top of the page, renaming indexes using the negative Christoffel term as a base gives

$$R^{a}{}_{bcd} = \Gamma^{a}{}_{bd,c} - \Gamma^{a}{}_{bc,d} + \Gamma^{a}{}_{c\rho}\Gamma^{\rho}{}_{bd} - \Gamma^{a}{}_{d\rho}\Gamma^{\rho}{}_{bc}$$

Right, now a result needs to be derived

By inspection, it can be seen that Riemann is antisymmetric in the last two indexes d and c

$$R^{a}{}_{bcd} = -R^{a}{}_{bdc}$$

By cyclic rotation of the last 3 indexes of Riemann we get

$$R^{a}{}_{bcd} = \Gamma^{a}{}_{bd,c} - \Gamma^{a}{}_{bc,d} + \Gamma^{a}{}_{c\rho}\Gamma^{\rho}{}_{bd} - \Gamma^{a}{}_{d\rho}\Gamma^{\rho}{}_{bc}$$

$$R^{a}{}_{dbc} = \Gamma^{a}{}_{dc,b} - \Gamma^{a}{}_{db,c} + \Gamma^{a}{}_{b\rho}\Gamma^{\rho}{}_{dc} - \Gamma^{a}{}_{c\rho}\Gamma^{\rho}{}_{d}$$

$$R^{a}{}_{cdb} = \Gamma^{a}{}_{cb,d} - \Gamma^{a}{}_{cd,b} + \Gamma^{a}{}_{d\rho}\Gamma^{\rho}{}_{cb} - \Gamma^{a}{}_{b\rho}\Gamma^{\rho}{}_{cd}$$

swap some index's in the last two, due to symmetry of the Christoffels

$$R^{a}{}_{dbc} = \Gamma^{a}{}_{dc,b} - \Gamma^{a}{}_{bd,c} + \Gamma^{a}{}_{b\rho}\Gamma^{\rho}{}_{dc} - \Gamma^{a}{}_{c\rho}\Gamma^{\rho}{}_{bd}$$

$$R^{a}{}_{cdb} = \Gamma^{a}{}_{bc,d} - \Gamma^{a}{}_{cd,b} + \Gamma^{a}{}_{d\rho}\Gamma^{\rho}{}_{bc} - \Gamma^{a}{}_{b\rho}\Gamma^{\rho}{}_{dc}$$

and adding these to our first Riemann gives

$$R^{a}{}_{bcd} + R^{a}{}_{dbc} + R^{a}{}_{cdb} = 0$$

but forget about this just for now

Back to our commentator, with the index names realigned to our Riemann definition

$$[\nabla_c, \nabla_d]x^a = (\nabla_c \nabla_d - \nabla_d \nabla_c)x^a = (V_{;cd} - V_{;dc})x^a = R^a{}_{bcd}x^b$$

We need to expand on this formula a bit in order to derive our acceleration of geodesics i.e. geodesic deviation.

Going back to our geodesic page, we noted

$$\nabla_{\mathbf{v}}\mathbf{V} \equiv \mathrm{V}^\alpha{}_{;\beta}\mathrm{V}^\beta$$

So now to work out the commentator of the above directional derivative

$$[\nabla_{\mathbf{w}}, \nabla_{\mathbf{v}}]\mathbf{V} = [w^\gamma \nabla_\gamma, v^\beta \nabla_\beta]\mathbf{V}$$

$$[\nabla_{\mathbf{w}}, \nabla_{\mathbf{v}}]\mathbf{V} = w^\gamma \nabla_\gamma v^\beta \nabla_\beta v^\alpha - v^\beta \nabla_\beta w^\gamma \nabla_\gamma v^\alpha$$

$$\begin{aligned}[\nabla_{\mathbf{w}}, \nabla_{\mathbf{v}}]\mathbf{V} = {} & w^\gamma v^\beta \nabla_\gamma \nabla_\beta v^\alpha - w^\gamma (\nabla_\beta v^\alpha)(\nabla_\gamma v^\beta) \\ & - v^\beta w^\gamma \nabla_\beta \nabla_\gamma v^\alpha + v^\beta (\nabla_\gamma v^\alpha)(\nabla_\beta w^\gamma)\end{aligned}$$

and swapping gamma with alpha in the 2nd product term

$$\begin{aligned}[\nabla_{\mathbf{w}}, \nabla_{\mathbf{v}}]\mathbf{V} = {} & w^\gamma v^\beta \nabla_\gamma \nabla_\beta v^\alpha - w^\beta (\nabla_\gamma v^\alpha)(\nabla_\beta v^\gamma) \\ & - v^\beta w^\gamma \nabla_\beta \nabla_\gamma v^\alpha + v^\beta (\nabla_\gamma v^\alpha)(\nabla_\beta w^\gamma)\end{aligned}$$

$$[\nabla_{\mathbf{w}}, \nabla_{\mathbf{v}}]\mathbf{V} = w^\gamma v^\beta \nabla_\gamma \nabla_\beta v^\alpha - v^\beta w^\gamma \nabla_\beta \nabla_\gamma v^\alpha - (\nabla_\gamma v^\alpha)(w^\beta \nabla_\beta v^\gamma - v^\beta \nabla_\beta w^\gamma)$$

$$[\nabla_{\mathbf{w}}, \nabla_{\mathbf{v}}]\mathbf{V} = w^\gamma v^\beta (\nabla_\gamma \nabla_\beta v^\alpha - \nabla_\beta \nabla_\gamma v^\alpha) - (\nabla_\gamma v^\alpha)(w^\beta \nabla_\beta v^\gamma - v^\beta \nabla_\beta w^\gamma)$$

$$[\nabla_{\mathbf{w}}, \nabla_{\mathbf{v}}]\mathbf{V} = w^\gamma v^\beta [\nabla_\gamma, \nabla_\beta] v^\alpha - (\nabla_\gamma v^\alpha)(\nabla_{\mathbf{w}}\mathbf{V} - \nabla_{\mathbf{v}}\mathbf{W})$$

$$[\nabla_{\mathbf{w}}, \nabla_{\mathbf{v}}]\mathbf{V} = w^\gamma v^\beta v^b R^\alpha{}_{b\gamma\beta} - (\nabla\mathbf{V})(\nabla_{\mathbf{w}}\mathbf{V} - \nabla_{\mathbf{v}}\mathbf{W})$$

but for

$$w^\alpha = \frac{\partial x^\alpha}{\partial \lambda} \text{ and } v^\beta = \frac{\partial x^\beta}{\partial \tau}$$

i.e. **w** is an affine parametized connecting vector and **v** is our velocity, the last term is zero, via

$$\nabla_{\mathbf{w}}\mathbf{v} - \nabla_{\mathbf{v}}\mathbf{w} = w^\alpha v^\beta{}_{;\alpha} - v^\beta w^\alpha{}_{;\beta}$$

$$\begin{aligned}\nabla_{\mathbf{w}}\mathbf{v} - \nabla_{\mathbf{v}}\mathbf{w} = {} & w^\alpha \left(\frac{\partial v^\beta}{\partial x^\alpha} + \Gamma^\beta{}_{\varepsilon\alpha} v^\varepsilon\right) \\ & - v^\beta \left(\frac{\partial w^\alpha}{\partial x^\beta} + \Gamma^\alpha{}_{\varepsilon\beta} w^\alpha\right)\end{aligned}$$

$$\nabla_{\mathbf{w}}\mathbf{v} - \nabla_{\mathbf{v}}\mathbf{w} = \frac{\partial x^{\alpha}}{\partial \lambda}\frac{\partial v^{\beta}}{\partial x^{\alpha}} + \Gamma^{\beta}{}_{\varepsilon\alpha} v^{\varepsilon} w^{\alpha}$$
$$- \frac{\partial x^{\beta}}{\partial \tau}\frac{\partial w^{\alpha}}{\partial x^{\beta}} - \Gamma^{\alpha}{}_{\varepsilon\beta} v^{\beta} w^{\varepsilon}$$

$$\nabla_{\mathbf{w}}\mathbf{v} - \nabla_{\mathbf{v}}\mathbf{w} = \frac{\partial v^{\beta}}{\partial \lambda} + \Gamma^{\beta}{}_{\varepsilon\alpha} v^{\varepsilon} w^{\alpha}$$
$$- \frac{\partial w^{\alpha}}{\partial \tau} - \Gamma^{\beta}{}_{\varepsilon\alpha} v^{\alpha} w^{\varepsilon}$$

where alpha and beta has been swapped in the last r term

$$\nabla_{\mathbf{w}}\mathbf{v} - \nabla_{\mathbf{v}}\mathbf{w} = \frac{\partial x^{\beta}}{\partial \tau \partial \lambda} + \Gamma^{\beta}{}_{\varepsilon\alpha} v^{\varepsilon} w^{\alpha}$$
$$- \frac{\partial w^{\alpha}}{\partial \lambda \partial \tau} - \Gamma^{\beta}{}_{\alpha\varepsilon} v^{\varepsilon} w^{\alpha}$$

where epsilon and alpha has been swapped in the last r term, thus the commentator of w and v is zero, therefor

$$[\nabla_{\mathbf{w}}, \nabla_{\mathbf{v}}]\mathbf{V} = w^{\gamma} v^{\beta} v^{b} R^{\alpha}{}_{b\gamma\beta}$$

Acceleration or Geodesic Deviation

The next task, is to show why the Riemann tensor determines the acceleration of the geodesics, i.e. why

$$\mathbf{a}_{w} = \frac{D^2 w^a}{D\lambda^2} = \mathbf{R}^{a}{}_{bcd} v^b w^c v^d$$

or equivalently

$$\mathbf{a} = \nabla_{\mathrm{v}}\nabla_{\mathrm{v}}\ \mathrm{w} = R^{a}{}_{bcd} v^b w^c v^d$$

To do this we need to show the following results, where D is the covariant derivative operator and λ is a fine parameter indeed, e.g. **x**=**x**(λ), t=t(λ).,

If we go back to our geodesic equation for acceleration, which sort of defines what is meant by acceleration in generalized co-ordinates.

$$\frac{d^2 x^{\mu}}{d\tau^2} + \Gamma^{\mu}{}_{\beta\gamma}\frac{dx^{\beta}}{d\tau}\frac{dx^{\gamma}}{d\tau} = 0$$

which can be written as

$$\nabla_{\mathbf{v}}\mathbf{V} \equiv \mathrm{V}^{\alpha}{}_{;\beta}\mathrm{V}^{\beta} = 0$$

So we can obviously write, actually this seems to pick something out of nothing, almost.

$$\nabla_{\mathbf{w}}\nabla_{\mathbf{v}}\mathbf{V} = 0$$

$$\nabla_{v}\nabla_{w}\mathbf{V} + \nabla_{w}\nabla_{\mathbf{v}}\mathbf{V} - \nabla_{v}\nabla_{w}\mathbf{V} = 0$$

$$\nabla_v \nabla_w \mathbf{V} + [\nabla_w, \nabla_v]\mathbf{V} = 0$$

$$\nabla_v \nabla_v \mathbf{W} + [\nabla_w, \nabla_v]\mathbf{V} + \nabla_v \nabla_w \mathbf{V} - \nabla_v \nabla_v \mathbf{W} = 0$$

$$\nabla_v \nabla_v \mathbf{W} + [\nabla_w, \nabla_v]\mathbf{V} + \nabla_v (\nabla_w \mathbf{V} - \nabla_v \mathbf{W}) = 0$$

but, from up above, the last term is zero so finally then, using our extended commentator formula

$$\mathbf{a} = \nabla_{\mathrm{v}} \nabla_{\mathrm{v}}\ \mathrm{w} = R^a{}_{bcd} v^b w^c v^d$$

and we seem to have lost a minus sign, so we'll leave that as an exercise for the reader.

Bianchi Identity and the Einstein Tensor

We have from above:

$$(V_{;cd} - V_{;dc}) x^a = R^a{}_{bcd} x^b$$

or

$$x^a{}_{;cd} - x^a{}_{;dc} = x^b R^a{}_{bcd}$$

Which means that taking the covariant 2nd derivative, in different orders, does not give the same result, as do ordinary derivatives.

It should be no surprise that, in the same manner as the covariant derivative itself, that

$$x_{a;cd} - x_{a;dc} = -x_a R^a{}_{bcd}$$

This can be seen from inspection from the initial derivation equation, and that again, just as in the covariant derivative case, where each tensor order index generates its own Christoffel symbol term, higher order tensors generate additional Riemann terms thus:

$$x^a{}_{e;cd} - x^a{}_{e;dc} = x^b{}_e R^a{}_{bcd} - x^a{}_b R^b{}_{ecd}$$

Now differentiate

$$x^a{}_{;cd} - x^a{}_{;dc} = x^b R^a{}_{bcd}$$

$$(x^a{}_{;cd} - x^a{}_{;dc})_{;e} = x^b{}_{;e} R^a{}_{bcd} + x^b R^a{}_{bcd;e}$$

Now set

$$x^a{}_e -> -x^a{}_{;e}$$

And replace into our 2nd term Riemann expression.

$$-(x^a{}_{;e})_{;cd} + (x^a{}_{;e})_{;dc} = -(x^b{}_{;e}) R^a{}_{bcd} + (x^a{}_{;b}) R^b{}_{ecd}$$

or

$$-x^a{}_{;ecd} + x^a{}_{;edc} = -x^b{}_{;e} R^a{}_{bcd} + x^a{}_{;b} R^b{}_{ecd}\ `$$

and bringing down from above

$$x^a{}_{;cde} - x^a{}_{;dce} = x^b{}_{;e} R^a{}_{bcd} + x^b R^a{}_{bcd;e}$$

Now a little bit of index swapping on the above two equations:

$$-x^a{}_{;ecd} + x^a{}_{;edc} = -x^a{}_{;e} R^b{}_{acd} + x^a{}_{;b} R^b{}_{ecd}$$

$$x^a{}_{;cde} - x^a{}_{;dce} = x^a{}_{;e} R^b{}_{acd} + x^b R^a{}_{bcd;e}$$

Now, if these last two equations are cycled in the last 3 index of Riemann and the 6 added together:

$$-x^a{}_{;ecd} + x^a{}_{;edc} = -x^a{}_{;e} R^b{}_{acd} + x^a{}_{;b} R^b{}_{ecd}$$

$$-x^a{}_{;dec} + x^a{}_{;dce} = -x^a{}_{;d} R^b{}_{aec} + x^a{}_{;b} R^b{}_{dec}$$

$$-x^a{}_{;cde} + x^a{}_{;ced} = -x^a{}_{;c} R^b{}_{ade} + x^a{}_{;b} R^b{}_{cde}$$

$$x^a{}_{;cde} - x^a{}_{;dce} = x^a{}_{;e} R^b{}_{acd} + x^b R^a{}_{bcd;e}$$

$$x^a{}_{;ecd} - x^a{}_{;ced} = x^a{}_{;d} R^b{}_{aec} + x^b R^a{}_{bec;d}$$

$$x^a{}_{;dec} - x^a{}_{;edc} = x^a{}_{;c} R^b{}_{ade} + x^b R^a{}_{bde;c}$$

Hence:

$$x^b (R^a{}_{bcd;e} + R^a{}_{bec;d} + R^a{}_{bde;c}) + (R^b{}_{ecd} + R^b{}_{dec} + R^b{}_{cde}) x^a{}_{;b} = 0$$

But the 2nd term is zero from the result up the page, somewhere…

Hence:

$$R^a{}_{bcd;e} + R^a{}_{bec;d} + R^a{}_{bde;c} = 0$$

This is the Bianchi identity that is needed in the construction of the Einstein Tensor.

Website last modified 31st May 2013

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