General Relativity For Tellytubbys

The Special Relativity Section

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Overview

Special Relativity (S.R.) has probably spawned more responses from individuals then a naked Heather Locklear strolling through central park (sci.physics.relativity). The main reason for this section is that it is required for an understanding of the stress-energy tensor.

You can get a bit more of an understanding for the rational of SR here SR Background.

So, straight to the point. It is noted that, independent of whether light is treated as a wave, particle or Teletubby, and independent of ones velocity with respect to the light source, the speed of light is invariant in vacume. That is, it is always measured, or all experiments can only be understood, if the speed of light always remains constant. This is of course very queer. No matter how fast one travels into the sun, its light will always be measured to have the same speed.

Secondly, various results of physics don't seem to depend on what does the moving, only relative velocities seem to matter.

This is summarized in the Einstein postulates.

1)Don't matter who's tits are moving, the results are always the same.

2) Invariable, when you switch the light on after a hard nights drinking, you wish you didn't

But these are more commonly expressed as:

1) All uniform motion is relative.

2) The speed of light, in vacume, is an invariant.

Postulate 1 is quite often expressed in more hi brow terminology which can be a bit confusing, so it is stated here in a form more suitable of all of us Teletubbys to understand. The essential bit is that it is just as valid to swap over what object is stationary and what object is moving. That is, there is no concept of absolute rest, all uniform motion is equivalent. In posh talk "uniform motion" is called "inertial motion", and things moving relatively to something else are described as moving in an inertial frame. It is important to point out there is absolutely nothing new in this at all from Newtonian physics. Einstein had f'all to do with this concept whatsoever.

Postulate 2 is often described, as the speed of light is a constant, the term "invariant" gives a much more accurate description of what the speed property of light really is.

The Sums of the Lorentz Transformation

Everyone got to the equations before Einstein, but somehow he managed to steal their thunder. The Lorentz Transforms were first published in Playboy, but by someone who, for obvious reasons, wanted to remain anonymous. So because of this Lorentz became the fall guy, and in fact he only became aquatinted with them whilst waiting for the number 133 bus at Dagenham East, while "reading" the news section, or so he claimed.

Here we go. Lets pretend to derive the co-ordinate transform equations of SR. Imagine standing in a box and viewing another a box moving by you, left to right (the x direction) at velocity v, that's our co-ordinate systems described then, i.e. where the origins are the front left corners of the boxes. The moving systems origin is called O', the stationary (reference) one O. If the reference frame emits a pulse of light from its origin, the pulse's position can be expressed by:

$$x^2 + y^2 + z^2 = c^2t^2$$

Since the pulse's radial position is ct

or

$$x^2 + y^2 + z^2 - c^2t^2 = 0$$

For the moving box, its equation for the light pulse would be:

$$\bar{x}^2 + \bar{y}^2 + \bar{z}^2 = c^2\bar{t}^2$$

or

$$\bar{x}^2 + \bar{y}^2 + \bar{z}^2 - c^2\bar{t}^2 = 0$$

Where, by assumption the t bar might be different, and it is to be determined if that is indeed the case. Note, our speed of light postulate requires that c is the same in both sets of equations.

These equations can then be equated thus:

$$x^2 + y^2 + z^2 - c^2t^2 = \bar{x}^2 + \bar{y}^2 + \bar{z}^2 - c^2\bar{t}^2$$

And, to cut a long story short, a bit of rationalization will indicate that the y's and z's will in both systems will be the same, thus:

$$x^2 - c^2t^2 = \bar{x}^2 - c^2\bar{t}^2$$

The bit to notice here is that both sides of the equation are an invariant. i.e. its value don't change with the inertial frames motion.

So, how are the x an x bars related. One assumes for all sorts of high-brow reasons, that the coordinates are related by a linear transform:

$$\bar{x} = \gamma(x - avt)$$

$$\bar{t} = \gamma'(t - \frac{bv}{c^2}x)$$

These might look a bit contrived, but this simply states that the x bar and t bar are linear functions of x and t, the constants are arbitrary and to be determined, and all come out in the wash.

Lets see what the relativity postulate gets us first.

Well, it means that we can simply swap over the x to x bar and t tot bar and change the sign of the velocity, thus

$$x = \gamma(\bar{x} + av\bar{t})$$

$$t = \gamma'(\bar{t} + \frac{bv}{c^2}\bar{x})$$

Now if we substitute the above pair of equations into the above, above x equation…

$$\bar{x} = \gamma(\gamma(\bar{x} + av\bar{t}) - av\gamma'(\bar{t} + \frac{bv\bar{x}}{c^2}))$$

equating the coefficients of t bar:

$$0 = \gamma' av - av\gamma'$$

hence

$$\gamma' = \gamma$$

Using this result and equating the coefficients of x bar:

$$1 = (\gamma - \frac{abv^2}{c^2}\gamma)\gamma$$

hence

$$\gamma = \frac{1}{\sqrt{1 - \frac{abv^2}{c^2}}}$$

now from above

$$x^2 - c^2t^2 = \bar{x}^2 - c^2\bar{t}^2$$

Substituting into the above gets us:

$$x^2 - c^2t^2 = \gamma^2(x - avt)^2 - c^2\gamma^2(t - \frac{bvx}{c^2})^2$$

$$x^2 - c^2t^2 = \gamma^2(x^2 - 2avxt + a^2v^2t^2) - c^2\gamma^2(t^2 - \frac{2bvxt}{c^2} + \frac{b^2v^2x^2}{c^4})$$

For this to be true for all x and t the coefficients of each power of x and t must be equal:

xt term,

$$0 = 2av - 2bv$$

or

$$a = b$$

x^2 term

$$1 = \gamma^2(1 - \frac{b^2 v^2}{c^2})$$

t^2 term

$$1 = \gamma^2(1 - \frac{a^2 v^2}{c^2})$$

Putting it all together gives us.

$$\bar{x} = \gamma(x - vt)$$

$$\bar{t} = \gamma(t - \frac{vx}{c^2})$$

$$where \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

And, one must stress the importance of the second term in the time equation. It's the relativity of simultaneity term that makes SR actually work. It says time is also a function of distance separation.

Time dilation

How does time change in the moving system relative to the stationary, one?

$$\frac{d\bar{t}}{dt} = \frac{d}{dt}(\gamma(t - \frac{vx}{c^2}))$$

$$\frac{d\bar{t}}{dt} = \gamma$$

Length Contraction

$$\frac{dx}{d\bar{x}} = \frac{1}{\gamma}$$

Standard Results

The assumption here is that the reader will get the main SR stuff from elsewhere, this is really a refresher bit just to form a reference for the GR bits and pieces.

Instead of the conventional 3D space and 1D-time descriptions, in SR objects are reformulated in one 4-D description. For example, 3 D velocities are replaced by a 4-d velocity vector called the 4 velocity, u.

4-position

$$\mathbf{X} = [ct, x, y, z] = x^\alpha \mathbf{e}_\alpha$$

4 - velocity

$$\frac{d\mathbf{x}}{d\tau} = \frac{dx^\alpha}{d\tau}, \alpha \neq 0,$$

where the 0'th term of **u** is time t. Tao is the proper time. Such that:

$$u^0 \equiv c\frac{dx^0}{d\tau} = c\frac{dt}{d\tau} = \gamma c$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} = \gamma\frac{dx}{dt} = \gamma v^\alpha$$, alpha = 1,2,3

or

$$\mathbf{u} = [c\gamma, \gamma\dot{x}, \gamma\dot{y}, \gamma\dot{z}]$$

4 - Momentum

$$\mathbf{p} = m\mathbf{u}$$

where it is noted that the 0^{th} momentum component is:

$p^0 = mc\gamma = \frac{E}{c}$ as $E = m\gamma c^2$ where $E \equiv KE + mc^2$, and have a deko at the E=mc section for more details on this.

Or

$$\mathbf{p} = [\frac{E}{c}, \gamma m v^x, \gamma m v^y, \gamma m v^z]$$

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http://www.kevinaylward.co.uk/gr/index.html

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