Analog Design
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Bipolar Amplifier Design
Part 1

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Abstract

This paper forms an introduction to the fundamental aspects of the design of bipolar transistor amplifiers. The intent is that reader should be able to gain a big picture view, such that many circuits can be reasonably appreciated by inspection.

This paper addresses the key points that one requires to effectively design analog circuits, and as such will not address much of the underlying physics of transistor operation, i.e. the "why". What is much important is "what" a transistor does, i.e. how it behaves, irrespective of why.

It is assumed that the reader has some basic math, including calculus, and has at least a cursory knowledge of circuits and components. However, for those without such pre-requisites, other then the actual *derivations* of some equations, there should be little else to impede the understanding of the material

It is noted that detailed calculations of transistor circuits are rarely required due to the availability of inexpensive Spice, e.g SuperSpice, simulation software. However, this software does not replace the analogue design engineers requirement to understand what he is doing! GIGO as they say.

In is also noted that the equations developed here are based on inspection for the most part. The generalized small signal equivalent circuit approach sometimes used for more complicated designs, will be addressed in another paper.

Transistor Basics

A bipolar transistor is a device with 3 terminals, collector, base and emitter. There are two types, NPN and PNP. This paper will address mainly NPN types, as simply reversing all of the current flows and terminal voltages will automatically explain PNP types.

Despite much literature that implies other wise, a transistor is a *voltage controlled device* not a current controlled device. This is a fundamental point that must be taken completely to heart if one desires to effectively design transistor level circuits. Applying a voltage to the base emitter terminals of a transistor, when there is a voltage across the collector emitter, will result in the flow of current through the collector to the emitter essentially independent of the collector emitter voltage. Therefor the transistor should be thought of as a *voltage controlled current source*. The key point is that the base emitter junction V/I must satisfy the basic diode equation. It is the total diode current that matters. What leaks out of the base (re combination) is essentially, irrelevant. The base terminal merely provides a convenient method of impressing a voltage at the base emitter junction.

There is of course, a corresponding base to emitter current (I_b) that flows when a voltage is applied to the base emitter (V_{be}), but this current might be said to flap in the wind. The ratio of collector current to base current is named h_{fe} or beta. This ratio is typically 50 to 500, but is rather indeterminate having variations over the collector current range, and from device to device. The emitter current is clearly $(1+ h_{fe}).I_b$, since it includes the base current as well. Most transistor circuits are designed such that the base current does not unduly effect circuit operation.

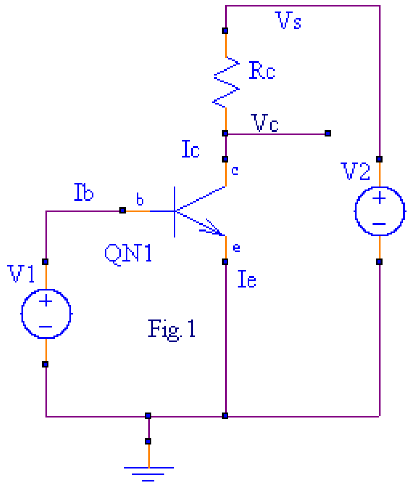

Fig. 1

The basic current equations for a transistor amplifier are therefore:

$$I_c \equiv h_{fe} I_b \qquad -1$$

$$I_e = I_c + I_b = h_{fe} I_b + I_b = (1 + h_{fe}) I_b \qquad -2$$

or from (1) and (2)

$$I_c = \frac{I_e h_{fe}}{(1 + h_{fe})} \qquad -3$$

or

$$\alpha \equiv \frac{I_c}{I_e} = \frac{h_{fe}}{(1 + h_{fe})} \qquad -4$$

Where it should be noted that as h_{fe} is of the order of at least 100, so that α is of the order of 0.99, hence the collector current is essentially equal to the emitter current. For most 1^{st} order design calculations, they are usually taken to be equal.

Transistor Bias

A transistor can only pass current in one direction when supplied by a single supply at its collector. This leads to the introduction of the concept of bias.

With reference to Fig.1, if V_{be} is increased from 0 to some value, current will flow through the collector. The initial voltage at the collector is equal to the supply voltage, Vs, as no current is drooped in Rc. The increase in current will cause an increase in the voltage dropped across Rc, hence the voltage at the collector will also drop. This means that the output goes in a negative direction when the input goes positive. This is usually referred to as phase inversion. However, if the input goes negative, the output can not raise any further then the supply voltage. The solution to this problem is to permanently "bias" the base emitter voltage with some fixed dc voltage, and then add the signal effectively in series with this voltage. This results in a fixed DC current always flowing through the collector, and also results in the collector being at some intermediate voltage between the power supply and zero. Now, for example, if the fixed DC voltage at the collector is say 5 volts, with a supply of 10 volts, the collector voltage can now swing above and below this DC "operating" condition.

It is noted that the use of the word "phase" in describing the inverted action of a basic transistor amplifier can be misleading, as it is not a phase shift in the exact sense. It is a voltage polarity inversion. However, for small signal, sine wave conditions (explained below), a polarity inversion is identical to a phase shift of 180 degrees. If the signal had say, a positive triangle and negative square shape, a 180 degrees phase shift would not be the same as the polarity inversion of a transistor amplifier.

Fundamental Transistor Equation

The fundamental equation describing bipolar operation is obtained from the classic diode equation, and is valid for diodes and the diode junctions of transistors.

$$I_e = I_o(e^{\frac{qV_{be}}{KT}} - 1) \quad -5$$

where K is boltzman's constant, q is the electronic charge and T is the absolute temperature of the device.

The quantity $V_t = KT/q$ is also usually defined in order to simplify the notation. At room temperature V_t is approximately 25mV.

I_o is named the reverse saturation current, (sometimes also called named I_s) and is a function of many internal parameters, including temperature. It is the current that will flow when the diode is reversed biased. This will be examined in a paper on the design of band gap voltage references.

With the above substation for V_t, the diode equation becomes:

$$I_e = I_o(e^{\frac{V_{be}}{V_t}} - 1) \quad -6$$

The -1 is usually dropped as it is insignificant for $V_{be} > 100mV$, except when leakage currents are of interest.

$$I_e = I_o e^{\frac{V_{be}}{V_t}} \quad -7$$

This equation is used in two ways.

1 Large signal conditions.

2 Small signal conditions

Large signal conditions are conditions such that the circuit is analyzed using complete F(V, I) equations or equations that are approximately correct for large voltage/current changes. Small signal conditions deal with the effects of small signal changes of current and voltage around some fixed large signal bias voltage or current. An example should make this a bit clearer.

Large signal conditions

Consider a 10 volt supply connected to a series resistor and diode. It can be deduced from the diode/transistor equation above that if the diode voltage changes by 60mv a 10:1 change will occur in the diode current. Thus the diode voltage don't change much!. Lets pretend then that if the diode has 0.75 volts across it at 100ma, then at 1A the diode voltage will therefor be 0.81. So it can be said that if the current in the circuit is somewhere between 100ma and 1A, the voltage across the resistor will be somewhere between (10-0.75) and (10-0.81) i.e. around 9.22 +/- a little bit. If we desire say, 500ma, letting R=9.22/.5 = 46ohms will set the current quite accurately, even though the diode voltage is not known precisely. i.e. it must be somewhere between 9.25/46 and 9.19/46. As an approximation therefore, it is often sufficient to consider that a diode produces a constant DC voltage drop irrespective of current.

Small signal conditions

Consider a small signal voltage i.e. say a signal of 10mV.Sin(wt) is added in series with the above circuit. The voltage that appears across the diode due to this signal will be reduced by the series resistor, and will *not* be constant in the context of small signals. The next section shows how this signal is calculat

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Transistor GM

The gm, or mutual conductance, of the transistor may be said to be the most fundamental design parameter of a transistor amplifier. It is expressed by:

$$I_e = gm.V_i \quad -8$$

That is, a small signal voltage at the base emitter such as 10mV.Sin(wt) will result in a small signal emitter current of I= gm.10mV.Sin(wt).

Another way of expressing this statement is by the reciprocal of gm which is named "re". That is, re=1/gm. Then:

$$I_e = \frac{V_i}{r_e} \quad -9$$

This small signal effective resistance, re, is a very useful concept. It states what the small signal voltage will be, if the small signal current through re is known. For the example above, this allows an easy calculation of the small signal voltage across the diode as $V_d = V_i$ (re/(R + re)) i.e. by the use of conventional formulas.

Derivation of Transistor GM

Since gm is the small change in current produced by a small change in voltage it is possible to approximate the gm by calculating a derivative from a general mathematical result:

$$\delta I \cong \frac{dI}{dV}\delta \mathrm{V} \quad -10$$

or

$$gm \equiv \frac{dI}{dV} \cong \frac{\delta I}{\delta \mathrm{V}}$$

From:

$$I_e = I_o e^{\frac{V_{be}}{V_t}} \quad -11$$

$$gm = \frac{dI_c}{dV_{be}} = \frac{d}{dV_{be}}(I_o e^{\frac{V_{be}}{V_t}})$$

$$gm = I_o \frac{1}{V_t} e^{\frac{V_{be}}{V_t}} = (I_o e^{\frac{V_{be}}{V_t}})\frac{1}{V_t}$$

$$gm = I_o \frac{1}{V_t} e^{\frac{V_{be}}{V_t}} = \frac{I_e}{V_t} = I_e \frac{q}{KT}$$

$$gm = I_e \frac{q}{KT} \quad -12$$

Hence the small signal gm is a function of the DC bias current. Too obtain a large gm you need large current, which of course means that a larger base current is required.

Plugging in the numbers for room temperature one gets:

$$gm = 40 I_e \quad -13$$

and

$$re = \frac{25mV}{I_e} \quad -14$$

So at 1ma, re =25ohms

Transistor Voltage Gain

The small signal collector current as a function of small signal input voltage (V_{be}) has now been found to be:

$$i_c = gmV_i \frac{h_{fe}}{(1+h_{fe})} \quad -15$$

When equation (3) is used to obtain the collector current from the emitter current

For simplicity, the hfe multiplier can be dropped by the assumption of large hfe, such that:

$$i_c = gmV_i \quad -16$$

There will be an output voltage developed across the collector resistor by V=-IR:

$$V_c = -gmR_cV_i$$

or

$$A_v = \frac{V_c}{V_i} = -gmR_c \quad -17$$

Which is often more usefully expressed as:

$$A_v = -\frac{R_c}{re} \quad -18$$

If an emitter resistor is added, the voltage to the base emitter will be reduced by Ic.Re, which is equivalent to having re increased by Re, since Ie flows through re, therefore:

$$A_v = -\frac{R_c}{(R_e + re)} \quad -19$$

So the way to think about transistor gain is by way of the load resister divided by effective emitter resistance.

It was noted above that at 1ma, re = 25 ohms so if the added emitter resistance is say 1k, re drops out.

<u>Mensa Test</u>

The voltage across the collector resistor of a transistor amplifier is 2V. What is the gain of the stage?

What bias current?. It don’t matter!

From (8) and (6)

$$A_v = \frac{V_c}{V_i} = gmR_c \quad -20$$

$$gm = 40I_c \quad -21$$

So:

$$A_v = 40 I_c R_c$$

But Ic.Rc is the DC voltage dropped across Rc so,

$$A_v = 40 V_{cdc} \quad -22$$

The gain is 80 then!

Second Order Effects rbb' and Ro

So far base current and non-ideal current output has been ignored. These effects of these will now be investigated.

Ro

It was stated that the transistor is a voltage controlled current source. The first correction to this is that there is a finite equivalent ac resistance across the collector emitter. This is noted by the fact that increasing the collector emitter voltage results in a small gradual increase of collector current. The change in Vce to the change in Ic is called it output resistance ro, or in the literature often described by $1/h_{oe}$. The effect of this resistance in a simple amplifier is to simply put it in parallel with the load resistor.

This output resistance is specified by a term named the "Early Voltage", or Va. It can be shown that the following holds in general:

$$r_o = \frac{Va + Vce}{I_c} \quad -23$$

That is, the small signal output resistance is a direct function of a constant and inversely related to the DC bias current. Typical Early voltages are from 30V to 200 volts.

If output characteristic plots are made, that is, one of Vce (x-axis) against Ic (y-axis), for various fixed base currents, all of the slopes of the Vce against Ic plots will intersect at the same negative Vce voltage. This magnitude of this voltage is the early voltage, hence why the ro slope is given by (21)

Often, the collector emitter voltage term is drooped, as the early voltage might be say, 100V so that with a collector emitter voltage of say, 5V, there is only 5% error.

$$r_o = \frac{Va}{I_c} \quad -24$$

This Early voltage leads to a maximum gain from a simple amplifier stage as follows.

Consider an ideal case where a perfect current source is the load on the transistor amplifier.

From (20)

$$A_v = gmR_c \quad = \frac{I_c}{V_t} r_o = \frac{I_c}{V_t}\frac{V_a}{I_c} \quad = \frac{V_a}{V_t} \qquad -25$$

For an Early voltage of 100V and a Vt of 25mV, then the maximum voltage gain is Av=4000.

Ways to bypass this limit with cascodes will be addressed in another paper.

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rbb'

There is base resistance from the base terminal to the real base of the transistor. This resistance is called the *base spreading resistor*. It varies slightly with collector current, but this effect will be ignored for now. Typically values for r_{bb}' are from 10 ohms to 500 ohms. This resistance can significantly effect noise performance.

The effect of this resistor is seen from inspection. Since the real base emitter voltage is reduced by ib.r_{bb}' which equals Ic.r_{bb}/h_{fe}, this value can simply be added in to our voltage gain equation:

$$A_v = \frac{R_c}{(R_e + re + \frac{rbb'}{h_{fe}})} \qquad -26$$

Other then for noise, and high frequency response, r_{bb}' can generally be ignored in many cases. For example, if Re was only 100, and h_{fe} was 200, a 50 base resistance might only effect the gain by (100 + 50/200)/100 = 1.0025.

Input Resistance

Given an input voltage it is obviously desirable to know what the resulting base current is. This is now calculated:

Equation (19) has to be corrected slightly as it was derived assuming that Ic was equal to Ie (equation (15). It is easy to see that Re and re need the prior dropped (1+h_{fe})/ h_{fe} term as a correction factor. Go back and check!

$$A_v = \frac{R_c}{(\frac{(R_e + re)(1 + h_{fe})}{h_{fe}} + \frac{rbb'}{h_{fe}})} \qquad -27$$

$$\frac{V_o}{V_i} = \frac{R_c}{(\frac{(R_e + re)(1 + h_{fe})}{h_{fe}} + \frac{rbb'}{h_{fe}})}$$

Rearranging

$$V_i = \frac{V_o}{R_c}(\frac{(R_e + re)(1 + h_{fe})}{h_{fe}} + \frac{rbb'}{h_{fe}})$$

$$V_i = I_c(\frac{(R_e + re)(1 + h_{fe})}{h_{fe}} + \frac{rbb'}{h_{fe}})$$

$$V_i = I_b h_{fe}(\frac{(R_e + re)(1 + h_{fe})}{h_{fe}} + \frac{rbb'}{h_{fe}})$$

$$V_i = I_b((R_e + re)(1 + h_{fe}) + rbb')$$

or

$$R_i = \frac{V_i}{I_b} = (rbb' + (R_e + re)(1 + h_{fe})) \quad -28$$

And approximately Ri $=h_{fe}$.Re'

Where Re' is the sum of Re and re

It is also shown later that Ri with Re=0, is often called h_{ie}

Emitter Follower Connection

This has the collector connected to the supply and the output taken from the emitter and emitter resistor junction. The following small signal equation can be seen from inspection.

$$V_i = V_{be} + I_e R_e$$

But V can be expressed by the gm or re equation previously derived, therefore

$$V_i = I_e r_e + I_e R_e$$

$$V_i = I_e(r_e + R_e) \quad -29$$

But

$$V_o = I_e R_e \quad -30$$

and dividing (30 by (29)

$$A_v = \frac{V_{oi}}{V_i} = \frac{I_e R_e}{I_e(r_e + R_e)} = \frac{R_e}{(r_e + R_e)} \quad -31$$

Hence the gain is near unity for large Re.

Writing (31) as

$$V_o = V_i \frac{R_e}{(r_e + R_e)} \qquad -32$$

Which is the same expression for a voltage source of V_i with internal impedance re driving a load Re. Hence, the emitter follower has an output impedance of re (in || with Re). So, although the voltage gain is only about unity, it has a low output resistance.

Base grounded, or Common Base Connection

In this configuration the input is at the emitter and the output is still taken from the collector. Vi.gm must still give the emitter current, as the signal is still applied between the same base and emitter! The gain must therefore also be the same, Rc/re. The difference in this connection is that the input resistance is only re, not h_{fe} re as the input signal now supplies all of the emitter current, not just a small fraction of it. The advantage of this connection is that it is often faster then the common emitter amplifier. This will be examined in another paper.

Summary of the main formulas

$$I_e = I_o(e^{\frac{qV_{be}}{KT}} - 1)$$

$$I_c \cong I_e$$

$$gm = I_e \frac{q}{KT}$$

$$gm = 40 I_e$$

$$re = \frac{25mV}{I_e}$$

$$i_e = gmV_i$$

$$r_o = \frac{V_E}{I_c}$$

$$r_e = \frac{1}{g_m}$$

$$A_{v\max} = \frac{V_E}{V_t}$$

Common Emitter

$$A_v = \frac{R_c}{(R_e + re)}$$

$$R_i = rbb' + (R_e + re)(1 + h_{fe}) \cong h_{fe}(R_e + re)$$

Emitter follower

$$A_v = \frac{R_e}{(r_e + R_e)} \qquad \text{-31}$$

$$R_0 = r_e$$

Common Base

$$R_i = r_e$$

$$A_v = \frac{R_c}{re}$$

The above formulas should be assimilated to heart. They are indispensable.

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