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# Relativistic Mass-Energy Equation

## Without the LT

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### Overview

The following constructs a derivation of relativistic mass without the Lorentz Transformation.

### Derivation

So, let's start with the well-known old stuff by the man who stood on the jolly green giant, to wit the relations of energy and momentum, thus:

$$F = \frac{d(mv)}{dt} \text{ - (1)}$$

$$KE = \int F dl \text{ - (2)}$$

If one assumes that the kinetic energy of a moving body is determined by any arbitrary function of velocity, and simply allow the Guinness mass to vary with velocity we have, in very general terms:

$$KE = k^2 m(v) + \alpha \text{ - (3)}$$

Where  $m(v)$ , the mass, is an arbitrary function of velocity,  $k^2$  and  $\alpha$  are arbitrary constants.

Therefore one can immediately write:

$$k^2 m(v) + \alpha = \int \frac{d(mv)}{dt} dl$$

And since  $\frac{dl}{dt} = v$ , one can write

$$k^2 m(v) + \alpha = \int v \frac{d(mv)}{dt} dt$$

Differentiating both sides of the equation w.r.t:

$$k^2 m'(v) = v \frac{d(mv)}{dt}, m' = \frac{d(m)}{dt}$$

$$k^2 m' = v^2 m' + vv' m, v' = \frac{dv}{dt}$$

$$k^2 m' \left(1 - \frac{v^2}{k^2}\right) = vv' m$$

$$\frac{m'}{m} = \frac{vv'}{k^2 \left(1 - \frac{v^2}{k^2}\right)}$$

This equation can be immediately integrated as the R.H.S. is an exact differential:

$$\ln(\beta m) = -\frac{1}{2} \ln\left(1 - \frac{v^2}{k^2}\right)$$

$$\therefore m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{k^2}\right)}} \quad (4)$$

So there you have it, no sweat at all. The form of the classic relativistic mass equation, without all that high-brow space-time posturing taken by some of those people that think they're clever, but are really just a pain in the arse to those of us that are.

Although it is not possible to show that  $k$  is  $c$ , the speed of light, in this derivation, we know its so cos Einstein said so, so it must be true.

And substituting back into to - (3), with suitable initial conditions will give:

$$KE = k^2 (m - m_0)$$

or

$$KE = k^2 \Delta m$$

And, cos we need this later for the "GR for Teletubbies" section we rearrange, with  $k=c$ :

$$mc^2 = KE + m_0 c^2$$

and with

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{k^2}\right)}}$$

then:

$$m_0 \gamma c^2 = KE + m_0 c^2$$

Now *define* the total energy, E by:

$$E_t = m_0 \gamma c^2$$

So that

$$E_t = KE + m_0 c^2$$

Identifying a rest energy term and a kinetic energy term. This bit is all rather a bit waffley really, but there you go...

And as a note, this concept of mass variation is really a little dated. Modern SR does not consider the mass to vary. Mass is an invariant. Momentum is redefined to achieve the same, but much more general effect as the result shown here.

### **Summary**

From these equations, all other relativistic equations can be derived, such as length contraction and time dilation. This direct derivation of the Einstein equation makes alternative theories quite dubious, in as much that the most fundamental, basic laws of Physics would have to be replaced, i.e. the definitions of Energy and Momentum.

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